Experiments on Bose-Einstein condensation

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1. Introduction

Bose-Einstein condensation (BEC) is a purely quantum phenomenon first predicted by A. Einstein in 1924 [1,2]. A gas of non-interacting identical particles described by a
wave function symmetric under the exchange of any two particles, should undergo a phase transition when the de Broglie wavelength becomes comparable to the spacing between particles. Under these conditions, the wave packets overlap and the indistinguishability of the particles becomes important. Particles described by a symmetric wave function are called bosons from S.N. Bose who was the first to study their statistical behaviour in the case of photons [3]. Bosons obey Bose-Einstein statistics. As is well known, the symmetrization postulate of quantum mechanics admits only another class of particles, those described by an antisymmetric wave function, which are called fermions and follow Fermi-Dirac statistics [4, 5]. If the temperature of a gas of bosons is lowered, below a transition temperature a macroscopic fraction of the particles should suddenly occupy the single lowest-energy quantum mechanical state. The condition for the transition is that \( n\Lambda_{dB}^3 = 2.612 \) [6], where \( n \) is the density of particles, and \( \Lambda_{dB} = \frac{h}{\sqrt{2\pi mk_B T}} \) is the thermal de Broglie wavelength, with \( h \) Planck’s constant, \( m \) the mass of particles, \( k_B \) Boltzmann’s constant, and \( T \) the gas temperature (the quantity \( n\Lambda_{dB}^3 \) is usually called phase-space density and a system of identical particles for which \( n\Lambda_{dB}^3 \sim 1 \) is referred to as a quantum-degenerate system). In fact, this prediction had never been tested directly although quantum degeneracy is at the origin of the behaviour of superfluid helium and exciton gases in semiconductors. A comprehensive review of experimental and theoretical work on BEC before 1995 can be found in [7].

In 1995, BEC was first observed directly by E. A. Cornell and collaborators in a gas of \(^{87}\text{Rb} \) atoms [8] and by W. Ketterle’s group in a gas of Na atoms [9]. Evidence of BEC in a gas of \(^{7}\text{Li} \) was also reported in [10,11]. These important results were achieved after several years of experimental efforts initially focused on atomic hydrogen. The observation of BEC in hydrogen was prevented by the presence of collisional processes leading to losses of atoms from the sample and by the difficulty in the detection of the atoms. In fact, the success of experiments of BEC on alkali atoms was the result of the combination of methods of magnetic confinement and evaporative cooling, mainly developed for hydrogen, with the techniques of laser cooling and trapping which are particularly efficient for alkali atoms. Also important was the occurrence of favourable collisional parameters and the possibility of easy optical diagnostics of the atomic sample.

This paper describes these recent experiments of BEC in dilute vapors of alkali atoms. Since the first observation of BEC, an amazing number of experimental and theoretical papers have been published. Because of this rapid development, it is impossible to give a complete bibliography and some of the results discussed here will necessarily become obsolete soon. An updated bibliography and information on recent results can be found at the Internet site http://amo.phy.gasou.edu:80/bec.html/. The purpose of this paper is to summarize the experimental methods that allowed the achievement of BEC, in several laboratories now, and to describe the main results obtained so far in the study of what can be considered as a new state of matter.

2. – Cooling and trapping of atoms

In recent years there was a dramatic progress in cooling and confinement of neutral atoms using electromagnetic fields. Cooling was achieved by using optical fields from laser sources (laser cooling) or by selective ejection of more energetic atoms from a trapping region using microwave radiation (evaporative cooling). Wall-free confinement of the cold atoms was accomplished by using laser beams (optical traps), by static or slowly varying magnetic fields (magnetic traps), or by a combination of the two (magneto-optical traps).

In this section we describe these cooling and trapping techniques with particular
emphasis on the methods which have been used so far to achieve Bose-Einstein condensation. The important “ingredients” of the experiments which have been successful so far are magneto-optical trapping, cooling in optical molasses, magnetic trapping, and evaporative cooling.

2.1. Laser cooling and trapping of atoms. The field of laser cooling and trapping of atoms has been one of the most active fields of research in physics in the past decade. Starting from the initial proposals that dated back to the seventies, several methods were developed which allowed to reach lower and lower temperatures. The study of the cooling processes became of interest in its own right as new mechanisms were discovered. In 1997, the Nobel Prize in Physics was awarded to C. Cohen-Tannoudji, S. Chu and W. D. Phillips for their important contributions in the development of methods to cool and trap atoms with laser light. It is beyond the scope of this review to describe in detail the different methods developed and the results obtained by the many groups in the world that have been involved in the research in this field. In this section, we only briefly describe the techniques of laser cooling and trapping that were important in the experiments in which Bose-Einstein condensation was achieved. A comprehensive review on both experimental and theoretical work in this field can be found in [12-16].

2.1.1. Slowing of an atomic beam. When an atom with mass $m$ absorbs or emits a photon with frequency $\nu$, because of conservation of total momentum its velocity changes by the recoil velocity $v_r = \frac{\hbar\nu}{mc} = \frac{\hbar}{mA}$. For the D$_2$ transition in sodium atoms, $\lambda = 589$ nm, $m = 23$ a.m.u., $v_r = 3$ cm/s. For cesium, $\lambda = 852$ nm, $m = 133$ a.m.u., $v_r = 3.5$ mm/s. This change in velocity is small but if several photons are scattered, a considerable change in the atomic velocity can result. For example, a sodium atom moving with a velocity $v_0 \sim 1000$ m/s interacting with a laser beam propagating in the opposite direction would stop after the absorption of $\frac{v_0}{v_r} \approx 33000$ photons. In the case of a thermal cesium beam ($v_0 \sim 300$ m/s) about 86000 photons are required to stop the atoms.

The absorption of each photon is followed by spontaneous emission. Because the emission direction is random, spontaneous emission has no effect, on the average, on the atomic velocity. The acceleration of the atom depends on the rate of scattering of photons. For high intensity of the light, an absorption-emission cycle requires a time of the order of the excited-state lifetime $\tau$. For the transitions considered above, $\tau \sim 20$ ns. Therefore atoms can be stopped in a few milliseconds.

In order to describe the process in more detail, a simple model can be considered of a two-level atom interacting with a plane light wave propagating in the direction $\mathbf{n}$ with frequency $\nu_L$ and intensity $I$ (fig. 1). The lower and upper states of the atom have energies $E_g$ and $E_e$, respectively, separated by the quantity $E_e - E_g = h\nu_A$.

The number of absorption-emission cycles in one second is given by the rate of spontaneous emission $1/\tau$ times the probability of occupation of the excited level, given by

$$p_e(\nu) = \frac{1}{2} \frac{I/I_0}{1 + (I/I_0) + (4/\Gamma^2)[\delta - (\nu_L/c)\nu \cdot \mathbf{n}]^2},$$

where $\delta = \nu_L - \nu_A$, $I_0$ is the saturation intensity, and $\Gamma = 1/(2\pi\tau)$ is the natural width of the atomic resonance.

The effect of the exchange of momentum between the atom and the radiation field is that the atom experiences a force $F_{sp}$ called spontaneous force or radiation pressure.
force. This force is given by the change of momentum of the atom in the unit of time, that is by the momentum of a photon times the number of absorption-emission cycles in the unit of time:

$$F_{sp}(v) = n \frac{\hbar \nu_L}{c} \frac{1}{2\tau} \frac{1}{2} \frac{I/I_0}{1 + (I/I_0) + (4/\Gamma^2)[\delta - (\nu_L/c)v \cdot n]^2}.$$  

Equation (2) gives the average force on the atom over several absorption cycles. This expression is valid if changes of the quantity \(\delta_{eff} = [\delta - (\nu_L/c)v \cdot n]^2\) can be neglected in the time interval considered.

Slowing of an atomic beam with a counterpropagating laser beam is efficient if the laser frequency is smaller than the atomic resonance frequency in order to compensate for the Doppler shift. As can be seen from eq. (2), the spontaneous force is most substantial when \([\delta + \nu_L(v/c)] \ll \Gamma\). For a given laser frequency, only atoms with a velocity within an interval of width \(\Delta v = \Gamma c / \nu_L\), called the capture range, will be slowed down. Figure 1 shows the change in the atomic velocity distribution produced by a counterpropagating laser beam with frequency \(\nu_L < \nu_A\). Atoms in the velocity capture range are slowed and their velocity distribution is narrower. Therefore, this scheme produces not only slowing but also cooling of the atoms. The problem of this scheme is that only a small fraction of the atoms interact with the laser. Also, the process stops when the atoms, because of the change in velocity, are no longer in resonance.
Two methods have been devised to solve these problems. The first method is called laser chirping [17]: The laser frequency $\nu_L$ is changed in time, typically in the form of a succession of ramps, so that groups of atoms see the laser in resonance until they reach the final velocity. The second method exploits a static space-varying magnetic field to tune the frequency $\nu_A$ [18]. The advantage of the laser chirping method is its simplicity that does not require particular changes in the atomic beam apparatus. It was further simplified by the recent advent of diode lasers whose frequency can be easily varied at a high rate. However, with this method, only "bunches" of atoms interact with the laser and eventually the slowed atoms are not localized but they are spread along the beam length. In the second scheme, instead, the nonuniform magnetic field can be designed so that almost every atom will see the fixed-frequency laser in resonance starting from a position which depends on the initial velocity. If the magnetic field is properly designed, a noticeable fraction of the atoms can be slowed down with this method and brought at a given velocity at the end of the magnet. The drawback of this scheme is that a large solenoid is required, whose shape must be accurately optimized. Also the presence of an intense magnetic field can represent a limit in some experiments.

The above methods are both used in present beam-slowing experiments and the choice of one method or the other depends on the particular requirements of the experiment.

2.1.2. Optical molasses. The possibility of cooling an atomic gas using laser radiation was first proposed by T. W. Hänsch and A. L. Schawlow in 1975 [19]. They pointed out that if a moving atom is irradiated with counterpropagating laser beams that are tuned slightly below the atomic resonance frequency, the Doppler shift will cause the atom to absorb preferentially photons moving opposite to its velocity (fig. 2). As discussed above, the atom will slow down. In a gas, this process will produce a cooling of the gas. The kinetic energy of the atoms is dissipated and converted into energy of the electromagnetic field.

If stimulated emission of photons is neglected, using eq. (2) the force on the atoms
can be written as the sum of the forces produced by each of the laser beams:

\[
F_{sp}(v) = \frac{h\nu_L}{c} \frac{1}{2\tau} \left[ \frac{I/I_0}{1 + (4/\Gamma^2)[\delta - (v/c)\nu_L]^2} - \frac{I/I_0}{1 + (4/\Gamma^2)[\delta + (v/c)\nu_L]^2} \right].
\]

In the limit of small atomic velocity \((v < \Gamma c/2\nu_L)\), the force can be written as

\[
F_{sp}(v) \sim \frac{h\nu_L^2}{c^2} \frac{8\delta}{\Gamma} \left[ \frac{I/I_0}{1 + (2\delta/\Gamma)^2} \right] v = -\alpha v.
\]

For low velocities, the force on the atoms depends linearly on velocity as in a viscous medium. This suggested the name optical molasses for this cooling configuration. The same scheme can be extended to three dimensions using three pairs of counterpropagating laser beams in three orthogonal directions. An optical molasses was first demonstrated experimentally by S. Chu and coworkers in 1985 with sodium atoms [20].

The minimum temperature expected for the atoms in optical molasses can be estimated, with the model considered above, taking into account two effects: The first is the cooling effect produced by the viscous force. The second is a heating of the atoms due to the random direction in which photons are absorbed and emitted. The temperature of the atomic sample at which these two effects balance is

\[
k_B T = \frac{h\Gamma}{4} \left( \frac{\Gamma}{2\delta} + \frac{2\delta}{\Gamma} \right).
\]

The minimum temperature is obtained for \(\delta = -\Gamma/2\) and is given by

\[
k_B T_D = \frac{h\Gamma}{2}.
\]

\(T_D\) is called Doppler limit. Typical values of \(T_D\) are 240 \(\mu\)K for sodium and 120 \(\mu\)K for cesium.

An important reference temperature for laser-cooled atomic gases is the so-called recoil temperature. Because the exchange of momentum between atoms and radiation takes place by discrete amounts, corresponding to the single photon momentum, the minimum spread of the atomic energies cannot be smaller than the energy corresponding to the single photon recoil. The recoil temperature \(T_r\) is therefore defined by

\[
\frac{1}{2} k_B T_r = \frac{1}{2m} \left( \frac{h\nu_L}{c} \right)^2.
\]

\(T_r\) is 2.4 \(\mu\)K for sodium and 197 nK for cesium.

2'1.3. Sub-Doppler temperatures in optical molasses. Accurate measurements of the temperature of the atoms in optical molasses, first performed by W. Phillips and coworkers by time-of-flight methods [21], showed that the temperature was much smaller than the expected Doppler limit temperature and not far from the recoil limit. This unexpected result was explained by J. Dalibard and C. Cohen-Tannoudji [22] and by S. Chu and coworkers [23]. They developed a theory of sub-Doppler laser cooling taking into account light-induced shifts of the atomic levels and optical-pumping effects.
A mechanism leading to sub-Doppler cooling can be qualitatively understood considering a 1-D model in which atoms having, for example, an angular momentum $J_g = 1/2$ in the lower state and $J_u = 3/2$ in the upper state interact with two counterpropagating laser beams with orthogonal linear polarizations. The polarization of the resulting field varies with position. Figure 3 shows the light-shifted atomic levels. The shift of a given Zeeman sublevel depends on the quantity $I/\delta$ and on the excitation probability for atoms in that level. Since this probability depends on the light polarization, the position-dependent polarization of the light leads to a position-dependent shift of the atomic levels. The combined effect of the spatially dependent light shift of the levels and of optical pumping is at the origin of the cooling mechanism. An atom starting, for example, at $z = \lambda/8$ in the $g_{-1/2}$ state and moving to the right, will climb the potential hill so that part of its kinetic energy will be converted into potential energy. If the velocity is sufficiently small, near the top of the hill the atom will be optically pumped into the $g_{+1/2}$ state. The potential energy gained at the expense of kinetic energy is dissipated in this spontaneous Raman anti-Stokes process. Optical pumping is then the mechanism allowing dissipation of energy and cooling of the atoms. This cooling effect was called Sisyphus cooling because, as in the Greek myth, the atoms are forced to continuously climb potential hills. The minimum temperature predicted by this model is of the order of the light-induced splitting of the levels. A linear dependence on $I/\delta$ is then expected.

Optical molasses are often realized with a light polarization scheme different from the one considered above. The two counterpropagating laser beams have opposite circular polarizations. Also in this case, sub-Doppler temperatures can be reached. The mechanism can be understood as follows: for two counterpropagating beams with opposite circular polarization and same amplitude, the resulting field is linearly polarized with the polarization vector rotating along the standing wave thus forming a helix with a pitch $\lambda$. It was shown in [22] that with this polarization configuration, the combined effect of the polarization rotation, light shift, and optical pumping produces a motion-induced
difference in the population of the ground-state Zeeman sublevels. This gives rise to an imbalance between the radiation pressures of the two beams and therefore to a friction force. This mechanism only works for $J \geq 1$ in the ground state.

The cooling mechanisms described above both depend on the presence of a light polarization gradient. However, since different processes lead to cooling in the two cases, different values for the friction and diffusion coefficients result. The steady-state temperatures are instead found to be about the same and to be proportional to the quantity $T/\delta$. This behaviour was confirmed experimentally for three-dimensional molasses down to temperatures of a few $\mu$K [24], corresponding to about ten times the recoil temperature.

2.1.4. Magneto-optical trapping of atoms. The magneto-optical trap (MOT) is nowadays a common method to confine and cool neutral atoms. In particular, it played a crucial role in BEC experiments. It was first proposed by J. Dalibard and demonstrated experimentally in [25].

A MOT consists of a combination of a set of laser beams and a quadrupole magnetic field. It is usually realized with six laser beams, which form three orthogonal pairs each made of two counterpropagating beams with opposite circular polarizations, intersecting in the zero point of the magnetic field generated by two parallel coils with currents flowing in opposite directions.

The basic mechanism can be understood, in the frame of a model similar to the one for Doppler cooling, by considering the 1-dimensional scheme shown in fig. 4. For simplicity, we consider an atom with a $J = 0$ ground state and a $J = 1$ excited state. The magnetic field, that is null at the center and has a magnitude proportional to the distance from the trap center, produces a position-dependent Zeeman splitting of the levels, as shown in the figure. If the laser frequency $\nu_L$ is smaller than the atomic resonance $\nu_A$ and the polarizations of the laser beams are chosen as indicated in the figure, atoms will experience a force towards the center of the trap. In fact, an atom displaced to one side with respect to the position where the magnetic field is null, will preferentially absorb photons from the laser beam coming from that direction and will be pushed towards the origin. As for optical molasses, the force on an atom can be written, in the low intensity limit, as the sum of the forces due to each of the beams. In this case, however, the Zeeman shift of the levels must be taken into account. In the limit of a small atomic velocity $v$ and a small Zeeman frequency shift $\kappa z$, the force can be written as

$$F_{sp}(v) \sim \hbar \frac{\nu_L}{c} \frac{8\delta I/I_0}{\Gamma} \left[ \frac{1}{1 + (2\delta/\Gamma)^2} \right] \left[ \frac{\nu_L}{c} \nu + \kappa z \right].$$

The motion of an atom is therefore that of a damped harmonic oscillator. The important characteristic of the MOT is that the atoms are not only confined but their kinetic energy is also dissipated as in atomic molasses. In fact, it was found [26] that sub-Doppler mechanisms due to polarization-gradient effects also play an important role in a MOT, leading to an increase of the confining force and to temperatures below the Doppler cooling limit. Atoms can be collected in a MOT from a slowed atomic beam as in [25] or from the room temperature vapor in a cell, as first demonstrated in [27].

The rate of change in the number $N$ of trapped atoms is given by

$$\frac{dN}{dt} = R - \gamma N - \beta \int n^2 dV,$$
Fig. 4. – Magneto-optical trapping. a) 1-dimensional scheme of the experimental set-up for a magneto-optical trap. b) Transition scheme. c) 1-dimensional model of the MOT.
where $R$ is the collection rate, $\gamma$ is the loss rate due to collisions of the trapped atoms with background gas, and the last term gives the loss rate associated with collisions between trapped atoms with a density $n$ and loss coefficient $\beta$. In the case of a vapor-cell trap, if the last term in eq. (9) is neglected, the steady-state number of trapped atoms is $N_s = R/\gamma$. The rate $R$ at which atoms with a velocity smaller than the capture velocity of the trap $v_c$ enter the volume $V$ defined by the intersection of the laser beams is [27]

$$\tag{10} R = \frac{nV^{2/3}}{2} \frac{v_c^4}{(\frac{m}{2KT})^{3/2}},$$

where $n$ is the density of the atoms in the vapor, $T$ is the temperature of the cell, and $m$ is the atomic mass. The capture velocity of the trap can be calculated as the maximum velocity for which an atom can be stopped within a distance equal to the laser beams diameter [28]. A rough estimate can be obtained considering a constant photon scattering rate $r = 1/2\tau_n$ where $\tau_n$ is the excited-state lifetime. The stopping distance for an atom with initial velocity $v$ is $l_{\text{stop}} = v^2/2rvr$, where $r_v$ is the recoil velocity. By imposing that the stopping distance equals the trapping beams diameter $d$, one gets

$$\tag{11} v_c = \sqrt{\frac{d}{\tau_n}}.$$

Equations (10) and (11) show that the loading rate $R$ is proportional to $d^4$. The loss rate $\gamma$ due to collisions of the trapped atoms with the hot atoms in the background vapor is given by $\gamma = n\sigma(3KT/m)^{1/2}$, where $\sigma$ is the collisional cross-section. The number $N$ of trapped atoms is then given by

$$\tag{12} N = \sqrt{\frac{\pi}{6}} \frac{d^4}{\tau_n^2} \frac{v_c^2}{\sigma} \left(\frac{m}{2KT}\right)^2.$$

In the case of a Cs MOT, $\tau_n = 30$ ns, $\sigma = 2 \times 10^{-13}$ cm$^2$ [27], $v_r = 3.5$ mm/s; if the laser beams have a diameter of 1 cm and the MOT is loaded from a room temperature vapor, one gets $N \approx 5 \times 10^9$ atoms which is of the right order of magnitude compared to what can be obtained experimentally in these conditions.

An important quantity, especially for BEC experiments, is the density of the trapped atoms. The maximum density of atoms that can be achieved in a MOT is limited by two main factors: first, the collisions between ground-state and excited-state atoms contributing to the $\beta n^2$ loss term in eq. (9) become important at the atomic densities typically obtained in MOTs; the second limit is due to repulsive forces between trapped atoms caused by reabsorption of scattered photons. Above a critical value of density, an increase of the number of trapped atoms gives a larger cloud with no increase of density. This represents also a limiting factor for the maximum number of atoms that can be confined in a MOT. In [29], a scheme called dark-spot trap was demonstrated that allowed a noticeable increase in the density of trapped atoms. The key idea is to isolate the cold atoms from the trapping light. This is obtained by optically pumping the atoms in the center of the trap into a state where they do not interact with the trapping laser. This scheme allows an increase of both the number of trapped atoms and their density. More than $10^{10}$ atoms can be confined in a dark-spot trap at densities of about $10^{12}$ atoms/cm$^3$. 
The maximum phase-space density obtained with the methods of laser cooling and trapping described above is in the range $10^{-5}-10^{-4}$.

2.1.5. Other schemes for laser cooling and trapping of atoms. In the effort towards the achievement of BEC in atomic gases, several methods have been developed to increase the phase-space density in a sample of atoms. Here, we only focus on the methods that were used in the experiments in which condensation was observed so far. In these experiments, a combination of optical cooling and trapping methods and of other techniques based on magnetic trapping and evaporative cooling was used. However, the possibility of reaching quantum degeneracy conditions with purely optical schemes cannot be excluded. Amongst the methods which look more promising in this prospect, are trapping by optical dipole traps and subrecoil cooling schemes such as Raman cooling and velocity-selective coherent population trapping.

The dipole trap is an optical trap based on the dipole force acting on an atom in the presence of nearly resonant light with a nonuniform intensity. This force is not caused by the scattering of photons due to spontaneous emission but can be considered as the effect of processes of absorption-stimulated emission of photons. In the dressed-atom picture, the dipole force arises from the spatially varying shift of atomic levels produced by a light field with a nonuniform intensity profile. For a laser frequency smaller than the atomic resonance frequency, the atoms will be attracted towards high-intensity regions. In contrast, for a positive detuning the atoms will be repelled from the high-intensity regions. A dipole trap was first realized in [30] using a single, red-detuned, focused laser beam. At the focus, the intensity has an absolute maximum such that three-dimensional confinement is possible.

Since no cooling mechanism is present in this trap, only atoms colder than the trap depth can be loaded into it. Also, the lifetime of the atoms in the trap is limited by the heating due to absorption and spontaneous emission of photons. The heating rate can be reduced by using large values of detuning for the trapping beam. Indeed, the photon scattering rate is proportional to $I/\delta^2$ while, for a large detuning, the trap depth depends on $I/\delta$. With the so-called far-off-resonance trap (FORT), trap depths of the order of mK can be obtained with lifetimes of several seconds [31]. Recently, a hybrid electro-optical dipole trap was demonstrated [32]; a static electric field was used to increase the confinement along the axial direction of the focused laser beam, that is a weak-confinement direction in the single-beam dipole trap. Blue-detuned dipole traps are also interesting and have been demonstrated [33]. In these traps, atoms can be confined by the repulsive walls created with the blue-detuned light and kept in regions where light is virtually absent.

Optical dipole traps have several positive features: they allow confinement of atoms at very high densities and in extremely small volumes. Contrary to the case of magnetic traps, the atoms in a dipole trap are not polarized. This can be of particular importance in experiments where mixed-spin systems are studied. Light traps can be quickly switched on and off. They can allow the confinement of atoms for which magnetic traps cannot be used. The main drawbacks of these traps are the high intensity required in order to use large values of detuning, and the need for good spatial stability of the trapping laser beam.

Of particular interest is the possibility of combining trapping in optical dipole traps and subrecoil cooling methods. Two cooling methods have been demonstrated leading to atomic samples where the velocity spread is smaller than the atom’s recoil velocity after emission of a photon: velocity-selective coherent population trapping (VSCPT) [34-36]
Fig. 5. – Energy of the Zeeman components of the hyperfine levels in the ground state of $^{87}$Rb and $^{85}$Rb as a function of the magnetic field.

and Raman cooling [37-40]. These cooling methods are both based on the idea of letting the atoms randomly scatter photons until they fall by spontaneous emission into a velocity-selective state which is no longer coupled to the laser field. In VSCPT, a c.w. light field tuned to a $J_g = 1 \rightarrow J_e = 1$ transition produces a velocity-selective dark state. This state is a linear superposition of magnetic substates having different linear momenta. In Raman cooling, on the other hand, the light field is pulsed. Atoms are pushed towards $v = 0$ with a sequence of laser pulses which induce velocity-selective Raman transitions followed by resonant excitation and spontaneous emission. Appropriate choice of laser pulse shape and detuning prevents atoms with $v \not= 0$ from being further excited. While VSCPT requires a specific type of atomic transition, Raman cooling can be applied to any three-level system with two long-lived states.

Raman cooling of atoms in a dipole trap was demonstrated in [41]. Sodium atoms were cooled down to a temperature of about $1 \mu$K with a density of $4 \times 10^{11}$ atoms/cm$^3$, corresponding to a factor $\sim 400$ in phase space density from BEC.

2'. Magnetic trapping. – Magnetic traps consist of an inhomogeneous magnetic field with a local minimum. Atoms with a non-zero magnetic moment $\mu$ in an inhomogeneous magnetic field $\mathbf{B}$ have an interaction energy given by $W = -\mu \cdot \mathbf{B}$ and experience a force $\mathbf{F} = \nabla (\mu \cdot \mathbf{B})$. The direction of the force depends on the orientation of the magnetic moment with respect to the field. If the atomic motion is adiabatic (the atom does not change Zeeman sublevel), it can be described by a local potential given by the atomic magnetic moment times $| \mathbf{B} |$. In the presence of a minimum of the magnetic field $| \mathbf{B} |$, atoms which are in a state whose energy increases with increasing magnetic field experience a restoring force towards the minimum region and can be trapped. Since for a static magnetic field only minima can exist in free space [42], only atoms in low-field-seeking states can be trapped. Figure 5 shows the energy of the Zeeman components of the ground state of $^{87}$Rb and $^{85}$Rb atoms as a function of the magnetic field. Magnetostatic traps work for $^{87}$Rb atoms in $| F = 1, m = -1 \rangle$ and $| F = 2, m = 1, 2 \rangle$ states and for $^{85}$Rb atoms in $| F = 2, m = -1, -2 \rangle$ and $| F = 3, m = 1, 2, 3 \rangle$ states.

A magnetic trap was first used by W. Paul to confine neutrons [43]. Magnetic trapping of neutral atoms was first demonstrated for Na [44]. More recently, magnetic traps were
used to confine H [45,46], Li [11], Na [47,48], K [49], Rb [50,51], Cs [27,52], and Eu [53].

Compared to magneto-optical traps, magnetic traps allow the confinement of the atoms at much lower temperatures because the effects due to radiation are absent (for example, the photon recoil limit). Magnetic traps are, however, much shallower than MOTs. The typical depth can be estimated considering a magnetic trap made of two parallel coils with currents flowing in opposite directions. As discussed in the following, such a configuration produces a field which is zero at the trap center and increases linearly around it. For a system of ~ 2 cm radius coils separated by ~ 3 cm carrying a current of ~ 500 At, the axial field gradient at the trap center is of the order of 100 G/cm and the lowest threshold is $\Delta B \sim 100$ G. Since the magnetic moment of a ground-state alkali atom is $\mu \approx \mu_B$, atoms can be trapped only if $k_B T < \mu_B \Delta B \approx 10^{-25}$ J, corresponding to a temperature of about 10 mK. Compared with atoms emerging from a thermal beam or a vapor at 300 K, it is clear that atoms must be pre-cooled before being loaded into a magnetic trap.

It must be noted that since magnetic traps are conservative traps, an independent mechanism is required to cool the atoms once they are in the trap. The methods demonstrated so far are evaporative cooling, which is described in detail in the next section, and sympathetic cooling via elastic collisions with another cold species in the trap (see sect. 3'6).

2.2.1. Quadrupole traps. The simplest scheme to realize a magnetic trap is the anti-Helmholtz configuration schematically shown in fig. 6a. This configuration produces a spherical-quadrupole magnetic field crossing a zero at the center and varying linearly around it. For two coils with radius $R$ separated by a distance $A$ and a current $I$ flowing, the field components in cylindrical coordinates are [54]

\[ B_z = 2cz, \quad B_\rho = -c\rho, \quad B_\phi = 0, \]

Fig. 6. – (a) Scheme of a quadrupole trap. (b) Scheme of a Ioffe-Pritchard trap.
where \( c = 3\mu_0 I AR^2 / 2(A^2 + R^2)^{5/2} \).

The potential energy for an atom in the trap is given by

\[
W = \mu |B| = \mu c\sqrt{4z^2 + \rho^2}.
\]

This type of trap was used in the initial experiments of atom trapping. In addition to the simplicity of the construction, it offers the advantage of a tight confinement of the atoms compared to other traps, discussed in the following, which are characterized by a parabolic minimum around a finite bias field.

A serious drawback of this trap becomes apparent, however, when it is used in combination with evaporative cooling. As the temperature of the atoms is lowered and their density increases, a loss of atoms from the trap is observed. This loss mechanism is due to the so-called Majorana spin-flips.

2.2. Losses due to Majorana spin-flips. As the atoms move in a magnetic trap, they experience a magnetic field changing direction in a complicated way. In order for the atoms to be trapped, the atomic magnetic moments must be oriented so that they are attracted towards the field minimum region. If the magnetic field changes slowly, the atomic magnetic moment precesses around the field and follows it adiabatically. The adiabaticity condition is violated if the Larmor frequency is smaller than the rate of change of the field direction. This is in fact the case for atoms passing close to the zero-field point in the center of a quadrupole trap. The probability of nonadiabatic transitions for a beam of oriented atoms crossing a region where the magnetic field goes to zero was calculated by E. Majorana [55]. Atoms in a quadrupole trap can undergo a spin-flip transition from one Zeeman sublevel to another and be lost from the trap [56]. The loss rate \( 1/\tau_0 \) caused by this effect can be estimated as follows [50]. If an atom with velocity \( v \) and mass \( m \) passes near the center of the trap with a minimum distance \( b \), it can undergo a nonadiabatic spin-flip if the Larmor frequency is smaller than the rate of change of the magnetic field direction \( v/b \). For a radial gradient of the field \( \partial B_r / \partial r = B'_q \), the Larmor frequency is \( \sim \mu B'_q / \hbar \). Loss then occurs within an ellipsoid of radius \( b_0 \sim (v\hbar / \mu B'_q)^{1/2} \). The loss rate is given by the flux through this ellipsoid, that is, the density of atoms \( N/l^3 \) times the area of the ellipsoid \( \sim b_0^2 \) times the velocity \( v \), where \( N \) is the number of trapped atoms and \( l \) is the radius of the atom cloud. The mean velocity and cloud size are related by the virial theorem: \( mv^2 \sim \mu l B'_q \). It follows that \( \tau_0 \sim (m/\hbar)^2 \). For \(^{87}\text{Rb} \) atoms, for example, \( m/\hbar \) is about \( 10^5 \text{s/cm}^2 \). The loss rate increases as the atoms are cooled and spend more time at the bottom of the trap near the zero-field point. In order to reduce losses due to spin-flips, different schemes were invented that prevent the atoms from crossing a zero-field region. They are briefly described in the following.

2.2.3. The time-averaged orbiting potential (TOP) trap. In the first experiment in which Bose-Einstein condensation was observed [8], a magnetic trap, named TOP trap, was used which had been invented trying to overcome the storage time limitation due to spin-flip losses while keeping the advantage of tight confinement of a quadrupole trap [50].

The idea of the TOP trap is to add to the quadrupole magnetic field a continuously changing bias field that moves the location of the field zero around much faster than the atoms can follow. In ref. [50] this was realized by adding to the static quadrupole field, produced by two horizontal coils in anti-Helmholtz configuration, a small magnetic field \( B_{\text{TOP}} \sim 10 \text{ G} \), rotating in the horizontal plane with a frequency \( \omega_{\text{TOP}} \) of the order of
10 kHz (fig. 7). The value of $\omega_{\text{TOP}}$ must be much smaller than the Larmor frequency for atoms in the bias field and much larger than the oscillation frequency of the atoms in the trap:

$$\omega_L \propto B_{\text{TOP}} \gg \omega_{\text{TOP}} \gg \omega_{\text{vib}}.$$  

The motion of the atoms is then governed by the time average of the instantaneous potential. It can be shown [50] that, using cylindrical coordinates with $z$ indicating the axis of the quadrupole trap, the resulting potential energy has the form

$$W_{\text{TOP}}(\rho, z) \simeq \mu B_{\text{TOP}} + \frac{\mu B_{\text{TOP}}^2}{4 B_{\text{TOP}}} (\rho^2 + 8z^2).$$

The trap is therefore harmonic to lowest order and the magnetic field in the center of the trap is $B_{\text{TOP}} \neq 0$.

2.2.4. The optical-plug trap. Another successful method to “plug the hole” leading to losses in quadrupole traps makes use of the optical dipole force to repel the atoms from the central region of the trap. This method was used in the first experiment in which Bose-Einstein condensation was observed in a gas of sodium atoms [9]. The optical plug was created by an Ar$^+$ laser beam tightly focused at the point of zero magnetic field. The blue-detuned far-off-resonance light produced a light-shift repulsive barrier that prevented the atoms from crossing the zero-field region in the trap. As for the other trapping schemes, the atoms were cooled by rf-induced evaporation. The potential experienced by the atoms is then a combination of the magnetic trapping potential, the repulsive potential due to the laser beam and the effect due to the rf. The resulting potential is shown in fig. 8.
2.2.5. The Ioffe-Pritchard-type traps. The trap called Ioffe-Pritchard (IP) trap was one of the first schemes to be suggested for trapping atoms [57]. It is similar to the Ioffe configuration introduced earlier for plasma confinement [58]. The basic scheme of this trap is shown in fig. 6b. It is made of two coils with parallel current, producing a so-called bottle field, and four straight conductors with current in alternating directions producing a quadrupole field for transverse confinement. If we use cylindrical coordinates with \( z \) indicating the axial direction, in proximity of the center the field components are [54]

\[
\begin{align*}
B_z &= c_1 + c_3(z^2 - \rho^2/2) + \ldots , \\
B_\rho &= -c_3z\rho + c_2\rho \cos(2\phi) + \ldots , \\
B_\phi &= -c_2\rho \sin(2\phi) + \ldots ,
\end{align*}
\]

where \( c_1, c_2 \) and \( c_3 \) are coefficients which can be calculated using the expressions for the field produced by a coil and by a straight conductor.

The resulting field \( |\mathbf{B}| \) is given by

\[
|\mathbf{B}|^2 = c_1^2 + 2c_1c_3z^2 + [c_2^2 - c_1c_3 - 2c_2c_3z \cos(2\phi)]\rho^2 + c_3^2(z^4 + \rho^4/4) + \ldots
\]

With proper choice of the configuration, the origin can be made a minimum for motion in any direction. The resulting potential is harmonic with a nonzero bias field at the origin:

\[
W = \mu \left[ c_1 + c_3z^2 + \frac{1}{2} \left( \frac{c_2^2}{c_1} - c_3 \right) \rho^2 \right].
\]

In this case, an atom passing near the center of the trap never finds a vanishing field and spin-flips can be suppressed. For this purpose, the Larmor frequency for atoms in the bias field must be larger than the vibrational frequency of the atoms in the trap:

\[
\omega_L \propto B_{IP} \gg \omega_{\text{vib}},
\]
where $B_{IP}$ is the bias field at the center of the trap (given by the $c_1$ term in eq. (17)). A bias field $B_{IP}$ of a few Gauss is typically sufficient.

In fact, in recent experiments of BEC, trap schemes that can be considered as variants of an IP trap were used. One is called "baseball" trap because it is made of coils following the pattern of the seams on a baseball [51]. Another scheme is called "cloverleaf" trap, again because of the shape of the coils [48]. These schemes will not be described in detail here for brevity.

2.2.6. Permanent-magnets traps. Intense fields and tight field curvatures can be easily obtained using permanent magnets. A trap made of permanent magnets was used in the experiment of BEC of $^7$Li [10, 11] (fig. 9). In this experiment, the trap oscillation frequencies were of the order of 150 Hz with a bias field of about 0.1 T. As discussed in subsect. 2.3, a large value of the field curvature, that is of oscillation frequency, makes evaporative cooling more efficient. On the other hand, permanent magnets have the disadvantage that the magnetic fields cannot be changed. This can be a serious limitation, for example, for the observation of the condensate.

2.2.7. Comparison of magnetic traps. Several factors must be taken into account for a comparison of the different schemes of magnetic traps for atoms. In experiments aiming to achieve BEC by evaporative cooling, it is important to reach the highest collisional rate after the atoms have been compressed in the magnetic trap while keeping the Majorana spin-flips to a negligible level. Important issues are also the simplicity of construction and the possibility of optical access. This latter considerations limited the use of traps made of superconducting magnets to hydrogen experiments where cryogenic systems are used.
If the geometric mean of the field curvatures is considered as a relevant figure of merit, a quantitative comparison can be made of TOP and IP traps [8,48]. The magnetic field gradient $B'$ and curvature $B''$ produced by a coil of radius $R_c$ at a distance $\sim R_c$ are

$$
(B')_z \simeq \frac{B_0}{R_c}, \quad (B'')_z \simeq \frac{B_0}{R_c^2} \simeq \frac{B'}{R_c} \simeq \frac{B'^2}{B_0},
$$

where $B_0$ is the field at the center of the coil. This gives a good estimate of the axial curvature in the IP trap. From eq. (18) it can be seen that the curvature of the radial field is given by

$$
(B'')_{x,y} \simeq \frac{B'^2}{B_{IP}},
$$

where $B_{IP}$ is the bias field at the center of the trap.

For the TOP trap, eq. (16) gives a curvature

$$
(B'')_{TOP} \simeq \frac{B'^2}{B_{TOP}}.
$$

The quantities to be compared are therefore

$$
(B''_{x,y})_{IP} \simeq \frac{B'^6}{B_0 B_{IP}^2}, \quad (B''_{x,y})_{TOP} \simeq \frac{B'^6}{B_{TOP}^3}.
$$

Equations (15) and (20) indicate that in order to avoid spin-flips, the minimum $B_{TOP}$ of a TOP trap must be much larger than $B_{IP}$ of an IP trap. Therefore for comparable field gradient $B'$, the radial curvature of the TOP trap is smaller than that of an IP trap (typically by two orders of magnitude). In the axial direction, instead, the curvature of the TOP trap is typically hundred times larger than that of an IP trap. The geometric mean of the curvatures is then usually better in an IP trap than in a TOP trap.

### 2.3. Evaporative cooling of trapped atoms.

Evaporative cooling of a gas of trapped atoms is based on the selective removal of atoms which have an energy higher than the average energy per atom and on rethermalization of the sample by collisions. Since the average energy of the atoms remaining in the trap is reduced in this process, the new equilibrium state of the gas corresponds to a lower temperature. As described in the following, methods have been demonstrated to force the cooling to proceed at a given rate. With optimized evaporative procedures, it is possible to achieve a dramatic reduction of the gas temperature and, more important for BEC experiments, an increase of the phase-space density by 5-6 orders of magnitude.

Evaporative cooling was first proposed [59] and demonstrated [60] in the effort to achieve BEC of atomic hydrogen. The technique was later extended to alkali atoms [50, 47] and, in combination with laser cooling, played a key role in the recent experiments on BEC.

Different methods of evaporative cooling have been demonstrated [61]. In the experiments on alkali atoms in which BEC was observed so far, rf-induced evaporation was used. An rf field was used to induce spin-flip transitions from trapped to untrapped states. The resonance frequency depends on the local magnetic field. Excitation can then
be energy-selective taking advantage of the connection between position and potential energy for the atoms in the trap (fig. 10).

In this subsection, we discuss some general features of the process and give the general scaling laws and the relevant parameters following essentially the presentation in [61]. Several detailed studies of evaporative cooling of trapped atoms have been published. An accurate overview on the subject can be found in [61, 62].

We consider a sample initially made of $N$ trapped atoms at a temperature $T$. The atoms are confined by a three-dimensional potential $U(r) \propto r^{3/\delta}$. Cases of particular interest for the present discussion are the linear potential ($\delta = 3$) and the parabolic potential ($\delta = 3/2$)(1). The average energy per atom is given by

\begin{equation}
E = \left(\frac{3}{2} + \delta\right) k_B T.
\end{equation}

As mentioned above, evaporative cooling works by “cutting the edge” of the confining potential so that high-energy atoms can escape. We indicate the depth of the resulting potential with $\epsilon_\ell = \eta k_B T$ and the average energy of the removed atoms with $(\eta + k) k_B T$. If $dN$ atoms escape from the trap, the energy of the $N' = N - dN$ atoms remaining in

---

(1) For an anisotropic potential, $U(x, y, z) = a \left| x \right|^{1/\delta_1} + b \left| y \right|^{1/\delta_2} + c \left| z \right|^{1/\delta_3}$ with $\delta = \delta_1 + \delta_2 + \delta_3$. 

---

Fig. 10. – Rf-induced evaporative cooling. (Figure courtesy of W. Ketterle.)
the sample is reduced by

$$dE = dN \left[ (\eta + k) - \left( \frac{3}{2} + \delta \right) \right] k_B T.$$  

Rethermalization by collisions leads to a new temperature $T' = T - dT$, given by the condition

$$N' \left( \frac{3}{2} + \delta \right) k_B T - dE = N' \left( \frac{3}{2} + \delta \right) k_B T'.$$

From eqs. (26), (27), neglecting second-order terms in $dN$ $dT$, it follows that

$$\frac{dT}{T} = \alpha \frac{dN}{N},$$

where

$$\alpha = \frac{\eta + k}{\frac{3}{2} + \delta} - 1.$$  

$\alpha$ is a crucial parameter in the evaporative cooling process, giving the temperature change per particle lost. In particular, if $\alpha$ is kept constant during the process, eq. (28) is easily integrated and gives

$$\frac{T_1}{T_2} = \left( \frac{N_1}{N_2} \right)^\alpha.$$  

Note that $\eta \gg 1 \rightarrow \alpha \gg 1$; eq. (30) shows then how efficient evaporative cooling can be. Equation (30) also shows that in the evaporation process $T$ is no longer independent of $N$. The scaling laws for the relevant physical quantities can then be expressed in terms of the number of atoms in the sample, by taking the dependence on $N$ and $T$ of these quantities for atoms in equilibrium in a confining potential and replacing $T$ by $N^\alpha$. The results are shown in table I. From the scaling laws in the table, it is possible to determine for which values of $\alpha$, and consequently of $\eta$, evaporative cooling works efficiently. A first consideration is that in order to have an increase of the phase-space density $D$ when the number of atoms in the trap is reduced, the exponent of $N$ in the expression of $D$ must be negative, that is

$$\alpha > \frac{1}{\delta + \frac{3}{2}}.$$  

Another condition to be fulfilled is that the evaporation does not lead to a reduction of the collisional rate $n \sigma_{\text{coll}} v$, which would slow down the process. The reduction of $v$ must be compensated by an increase of the density $n$. Table I shows that this is the case if

$$\alpha > \frac{1}{\delta - \frac{1}{2}}.$$  

This regime is called "runaway evaporation". Conditions (31) and (32) indicate that efficient evaporation requires a value of $\eta$ as large as possible. This is in agreement
TABLE I. – Dependence of relevant physical parameters on the number of trapped atoms $N$ during evaporation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature, $T$</td>
<td>$\propto N^\alpha$</td>
</tr>
<tr>
<td>Volume, $V$</td>
<td>$\propto N^{\alpha \delta}$</td>
</tr>
<tr>
<td>Density, $n$</td>
<td>$\propto N^{(1-\alpha \delta)}$</td>
</tr>
<tr>
<td>Average velocity, $v$</td>
<td>$\propto N^{\alpha/2}$</td>
</tr>
<tr>
<td>Elastic collision rate, $n_o a v$</td>
<td>$\propto N^{[1-\alpha(\delta-\frac{1}{2})]}$</td>
</tr>
<tr>
<td>Phase space density, $D$</td>
<td>$\propto N^{[1-\alpha(\delta+\frac{3}{2})]}$</td>
</tr>
</tbody>
</table>

with the intuitive argument that the larger the energy of the evaporating particles, the larger the temperature change. On the other hand, a large value of $\eta$ implies a low rate of evaporation. If the evaporation time is too long, other mechanisms such as collisions or Majorana spin-flips can produce losses of atoms from the trap. Optimization of the evaporative cooling then requires a compromise between efficiency and speed of the evaporation process taking into account the unavoidable loss mechanisms. It is beyond the scope of this review to analyze this problem in detail. An analytical expression for the rate of evaporation can be easily obtained, however, following an argument given in [61]. This rate can then be compared with the rate of losses of atoms from the trap. The argument is the following: For a large value of $\eta$, the rate of evaporation of the atoms is the same as the rate of production of atoms with energy larger than $\eta k_B T$. The evaporation rate can be written as the number of atoms with energy higher than $\eta k_B T$ times the elastic collision rate $1/\tau_{el}$. The number of atoms with $E > \eta k_B T$ in the untruncated Boltzmann distribution can be approximated with $2N \sqrt{\pi} \eta \exp[-\eta]$. The elastic collision rate is given by $n_o \sigma v$, where $n_o$ is the density of the particles, $\sigma$ is the elastic collision cross-section, and $v = \sqrt{2\eta k_B T/m} = \sqrt{\frac{\pi \eta \delta}{2}}$ is the velocity of atoms with energy $\eta k_B T$. The rate of evaporation can therefore be written as

$$\dot{N} = -n_0 \sigma \bar{v} \eta e^{-\eta} = -\frac{N}{\tau_{ev}}.$$  

Equation (33) gives the rate of change of the number of atoms due to evaporation. The loss of atoms due to background gas collisions or other loss mechanisms can be taken into account by considering an exponential decay of the number of atoms with a time constant $\tau_{loss}$. The rate of change of the collisional rate can then be written as

$$\frac{1}{n \sigma v} \frac{d}{dt}(n \sigma v) = -\left[\frac{1 - \alpha(\delta - \frac{1}{2})}{\tau_{ev}} + \frac{1}{\tau_{loss}}\right].$$

In order to have a constant or increasing collisional rate during evaporation, the term in the square bracket must be null or negative, that is

$$\frac{\tau_{ev}}{\tau_{loss}} \leq \alpha \left(\delta - \frac{1}{2}\right) - 1 \rightarrow \frac{\tau_{loss}}{\tau_{el}} \geq \frac{\tau_{ev}}{\tau_{el}} \alpha \left(\delta - \frac{1}{2}\right) - 1.$$  

The quantity $\tau_{ev}/\tau_{el}$ can be easily calculated in the limit of large $\eta$ considered above,
giving

\[
\frac{\tau_{ev}}{\tau_{el}} \approx \frac{\sqrt{2}e^\eta}{\eta}.
\]

For arbitrary values of \(\eta\), this ratio can be written in terms of generalized gamma functions.

Equation (35) shows that efficient evaporative cooling requires the rate of elastic collisions to be much larger than the rate of collisions causing losses of atoms from the trap (typical numbers in the experiments are \(\tau_{loss} \approx 200 - 500 \times \tau_{el}\)).

Two kinds of collisions are responsible for atom losses from magnetic traps: collisions with background gas and inelastic collisions. Losses due to background gas collisions can be reduced by reducing the pressure of background gas. Ultra-high vacuum chambers are indeed used in the experiments. Inelastic collisional processes can be either binary collisions, such as dipole relaxation and spin relaxation, or three-body recombination. Since evaporative cooling is based on collisions, a high density is required; inelastic collisions are then unavoidable. The situation is however quite different for different atoms depending on their collisional parameters. The success of BEC experiments on alkali atoms, compared with the experiments on hydrogen, is due to the fact that alkali atoms have a much larger elastic cross-section. Since the rate of inelastic collisions is roughly the same as in hydrogen, the ratio of "good" to "bad" collisions is much larger for alkali atoms. An analysis of the relevance of different kinds of collisions for different atoms is beyond the scope of this review. A comprehensive treatment of the subject can be found in [61] and references therein.

3. - Studies of Bose-Einstein condensates

3'1. Experimental procedure to achieve BEC. - In the previous section, the ingredients of BEC experiments were presented, that is the methods to cool and trap the atoms. Here, we describe the way these ingredients were put together in the first experiments in which BEC was observed. Table II gives a synthetic view of the general scheme of a BEC experiment with the possible variations demonstrated in the different laboratories. The procedure to observe BEC can in fact be divided into four main steps.

The first step is laser cooling and trapping of the atoms. This was accomplished by using a single vapor-cell MOT, or a laser-slowed atomic beam, or a double-MOT apparatus in which the atoms are loaded in a first MOT from the vapor and then transferred into a second, high-vacuum, chamber. In the first experiments, a dark-spot MOT was used to increase the atoms density. A molasses phase can be used to reduce the temperature of the atoms before loading them into the magnetic trap. In this first step a large gain in phase-space density is obtained. For example, the phase-space density in a room temperature vapor of Rb at a pressure of \(10^{-10}\) Torr is of the order of \(10^{-20}\). After laser cooling and confinement, a value in the range \(10^{-6}-10^{-5}\) can be reached.

The second step is the magnetic trapping of the atoms: This is the aspect in which the different experiments have more differentiated. As described in subsect. 2'2, several schemes have been developed to achieve a tight confinement of the atoms and to avoid losses due to Majorana spin-flips. The trap configuration must also be suitable for evaporative cooling of the atoms in the trap. An important stage in this phase is the adiabatic compression of the trapped atoms. After loading, the atoms in the trap are adiabatically compressed by increasing the trap curvature. This leads to an increase
Table II. - *Experimental procedures used to study Bose-Einstein condensation in atomic gases.*

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Method/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser cooling and trapping</td>
<td>Single vapor-cell (JILA) [8]</td>
</tr>
<tr>
<td></td>
<td>Slowed atomic beam (MIT, Rice) [9,48,11]</td>
</tr>
<tr>
<td></td>
<td>Double-MOT (JILA) [51]</td>
</tr>
<tr>
<td>Magnetic trapping</td>
<td>TOP (JILA) [8]</td>
</tr>
<tr>
<td></td>
<td>Optical plug (MIT) [9]</td>
</tr>
<tr>
<td></td>
<td>Permanent magnets (Rice) [11]</td>
</tr>
<tr>
<td></td>
<td>Cloverleaf (MIT) [48]</td>
</tr>
<tr>
<td></td>
<td>Baseball (JILA) [51]</td>
</tr>
<tr>
<td>rf-induced evaporative cooling</td>
<td>(JILA, MIT, Rice)</td>
</tr>
<tr>
<td>Optical imaging</td>
<td>Absorption imaging (JILA, MIT) [8,9,48,51]</td>
</tr>
<tr>
<td></td>
<td>Dark-ground imaging (MIT) [63]</td>
</tr>
<tr>
<td></td>
<td>Polarization imaging (Rice) [11]</td>
</tr>
</tbody>
</table>

of the temperature and density of the atoms, with the phase-space density remaining constant. The increased rate of elastic collisions obtained in this way is important to achieve the necessary conditions of rethermalization in the evaporative cooling process.

The third important stage is indeed evaporative cooling. In all of the experiments where BEC was obtained so far, atoms in the magnetic trap were cooled with evaporative cooling by using rf-induced evaporation. As mentioned in subsect. 2.3, the magnetically trapped atoms are irradiated with rf radiation. The rf frequency is tuned near the resonance frequency for transitions between nearby Zeeman sublevels. This produces spin-flip transitions from trapped to untrapped states. Since higher-energy atoms can reach higher-field regions of the trap, spin-flip resonance frequencies are more shifted than for colder atoms. By varying the frequency of the rf field, colder and colder atoms are eliminated from the trap. As discussed in subsect. 2.3, if the right procedure is followed, this process can give an increase of the phase-space density of several orders of magnitude and eventually allows to reach the condition for BEC (phase-space density ~ 1).

Once conditions for BEC have been reached, a proper observation method must be used. For this fourth stage, the methods demonstrated so far are based on absorption imaging and phase-contrast imaging. The basic idea and experimental realization of these detection methods are described in the next subsection.

In order to give a specific example of an experimental procedure to reach BEC, we can describe the way followed in the first, now almost “historical”, realization at JILA [8]. About $10^7$ $^{87}$Rb atoms were first collected in a dark-spot MOT from the room temperature vapor in a glass cell. Because of the very low pressure of Rb in the cell ($10^{-11}$ Torr), the loading of the MOT took about 300 s. The trapped atoms were compressed and cooled to 20 μK by adjusting the field gradient and laser frequency. The laser beams were then switched off and the magnetic trap (a TOP trap in this experiment), was quickly switched on. The magnetically trapped atoms were compressed by adiabatically increasing the quadrupole field. This produced a sample of $4 \times 10^6$ atoms in the trap with
a temperature of about 90 $\mu$K and an average density of about $2 \times 10^{10}$ atoms/cm$^3$. The trap had an axial oscillation frequency of 120 Hz. Evaporative cooling was then applied for about 70 s. During this time, the rf frequency was ramped down from the initial value to the final value that determined the final temperature. At the end of the evaporative cooling, the magnetic trap was switched off, letting the atoms expand freely. After 60 ms of expansion, the cloud was observed using an absorption imaging technique. The atoms were irradiated with a resonant laser beam for 20 $\mu$s and the shadow produced by the atoms was imaged on a charge-coupled-device (CCD) array, as described in the following.

3.2. Observation of a Bose condensate. – As mentioned above, in the experiments performed so far, at the end of the cooling process the atoms distribution was observed by imaging with an optical probe. Two major approaches have been followed: The first is absorption imaging of the sample, the second is based on the detection of refractive effects caused by the atoms.

Absorption imaging was used, for example, in [8, 9]. The atoms were illuminated with a short pulse of resonant light and the shadow of the cloud was imaged onto a CCD detector (fig. 11). Because of the high optical density ($D \approx 100$–300) of the atom cloud, however, quantitative measurements could not be performed by direct imaging of the atoms in the trap. At the end of the evaporative cooling, the cloud was allowed to expand by switching off the trapping fields and the absorption image was recorded after a time of several milliseconds. The larger size of the sample after the expansion also reduces spatial resolution requirements for the optical system. A result is shown in fig. 12. This method of observation corresponds to a 2-dimensional time-of-flight measurement of the
Fig. 12. - False-color images showing the velocity distribution of a sample of cold $^{87}$Rb atoms. Left: before condensation, center: just after the transition, right: for a nearly pure condensate. Images were recorded by the absorption imaging method after releasing the atoms from the trap and letting them expand freely. (Figure courtesy of E. A. Cornell.)

velocity distribution. The recorded image gives the initial velocity distribution of the atoms in the trap. However, for a harmonic confining potential, the spatial distribution and the velocity distribution are the same. From a single image, information can then be extracted on the density and the temperature of the atoms in the trap. This method is obviously a destructive observation method because each atom scatters several photons while being probed, heating the gas. Experiments are therefore performed by repeated cycles of load-evaporation-probing.

An alternative method to observe the condensate was demonstrated in [63]. By using a so-called dark-ground imaging technique [64], the dispersively scattered light was observed. In the dark-ground imaging technique, a collimated probe beam is sent through a weakly absorbing sample (fig. 13). The coherently scattered light is collected with a lens system and imaged onto a camera. The probe beam is blocked after passing the sample by inserting a small opaque object at the position where the beam comes to a focus. If $E_0$ is the electric field of the incident probe beam, after passing through the sample it acquires a spatially dependent phase $\beta = \varphi + i\alpha/2$ so that the transmitted electric field is given by $E = E_0 e^{i\beta}$. When the opaque object is inserted to block the incident beam, the resulting intensity is given by $I_s = E_0^2 \mid e^{i\beta} - 1 \mid^2$. In the case $\alpha \ll \varphi \ll 1$, the resulting signal is $I_s \approx E_0^2 \varphi^2 = I_0\varphi^2$.

Dispersive imaging offers two important advantages with respect to absorption imaging. First, because the signal depends on the phase shift $\varphi$, it is possible to detune the frequency of the probe light from resonance at values for which the optical density is
small. This allows probing of dense clouds of atoms directly in the trap. Second, disperse imaging can be a nondestructive detection method. In [63], it was demonstrated that by probing the condensate with a pulse of off-resonant light, several consecutive images of the same condensate could be taken with no observable deterioration of the signal (fig. 14). Indeed, in dispersive scattering the photons are elastically scattered by a small angle. Also, if a light pulse duration longer than the oscillation period in the trap is used, no momentum is transferred to the atoms on the average.

When the phase shift $\varphi$ is small, the dark-ground signal is small because it depends on $\varphi^2$. A signal linear in $\varphi$ can be obtained by a phase-contrast imaging method [11,65]. In situ imaging allowed the MIT group to observe the propagation of sound in a condensate,
to study the formation of the condensate and to directly observe the collective excitations. In [11], the birefringence of the sample of atoms polarized in the trap magnetic field was probed using a linearly polarized laser beam. After passing through the atomic sample, the polarization of the probe beam was analyzed with a polarizer. The intensity of the transmitted light was measured with a photodetector. This detection method allowed \textit{in situ} imaging of condensates with only $\sim 10^3$ atoms.

Some remarks can be made concerning dispersive imaging detection methods. As discussed above, they can be considered as nondestructive detection methods. In fact, the phase of the condensate is changed under the effect of light. Only the number of atoms in the condensate can be measured in this way. Also, one could expect light scattering to be affected by the quantum degenerate state of the atoms. Such effects are negligible, however, under typical experimental conditions [66].

\section*{3.3. Measurement of energy and ground-state occupation as a function of temperature}

Images of the ultra-cold atoms as the one shown in fig. 12, obtained by resonant absorption imaging, contain a wealth of information about important thermodynamic quantities. The integrated area under the distribution gives the total number, $N$, of atoms in the sample. The number of atoms in the ground state of the system, $N_0$, can be obtained from the integrated area under the narrow feature centered on zero velocity. The mean square radius of the expanded cloud as a function of time gives the mean square velocity, that is average energy, of the atoms. The temperature, $T$, can also be extracted from the analysis of the images. From the data on mean energy as a function of temperature, the specific heat can be determined, that is an important quantity in the study of phase transitions. At high densities of condensed atoms, the repulsive mean-field energy becomes important and can be measured.

In the work reported in ref. [67], a series of images of ultra-cold clouds of $^{87}$Rb atoms were analyzed to extract the ground-state occupation and the mean energy as a function of temperature. The number of atoms in the ground state was defined as the number of atoms contributing to the central feature in the optical-depth images. The condensate shape was fit with a 2-d Gaussian. The width, aspect ratio and peak height were found to be functions only of the number of atoms in the feature. The procedure gave consistent values of $N_0$ for temperatures down to about one half of the transition temperature. The temperature was determined by fitting a Gaussian to the high-energy tail of the velocity distribution. This procedure is based on the assumptions that the high-energy part of the velocity distribution remains in thermal equilibrium with the central part of the cloud and that it can still be described by a Maxwell-Boltzmann distribution because the density of the atoms in the outer part of the cloud is lower. Figure 15 shows the data obtained for the ground-state fraction $N_0/N$ as a function of scaled temperature $T/T_0$. The scale temperature, $T_0$, is the critical temperature for a noninteracting gas in the thermodynamic limit. The solid line in the figure shows the behaviour expected for the ideal gas of bosons in an anisotropic potential in the limit of infinite number of atom: $N_0/N = 1 - (T/T_0)^3$. The dotted line shows the curve calculated including finite-number corrections [68, 69]. The dashed line is, instead, a least-squares fit of the data with the function $N_0/N = 1 - (T/T_c)^3$, with $T_c$ as a parameter. The fit gives a value for the critical temperature $T_c = 0.94(5)T_0$. The accuracy of the data is however too low to show a difference between $T_c$ and $T_0$. An accurate measurement of the critical temperature would allow to test theories taking into account mean-field [70, 71] or many-body [72] interaction effects.

From the same images analyzed to measure the ground-state fraction, information
on the energy and specific heat was obtained. The specific heat is usually defined as the temperature derivative of the energy per particle at constant pressure or constant volume. In this case it is given by the slope of the scaled energy vs. temperature plot with the confining potential held constant. The result is shown in the inset of fig. 16 and compared with the specific heat of other relevant systems. The data indicate the presence of a step in the specific heat near the empirical transition temperature. The observed step is smaller than the one predicted by a finite-number, ideal-gas theory [68, 69]. The difference can be explained as an effect of interactions.

Fig. 15. Total number of atoms (inset) and ground-state fraction as a function of scaled temperature $T/T_0$. (From J. R. Ensher et al. (1996), with permission.)

Fig. 16. – Specific heat as a function of scaled temperature $T/T_0$ for various theories and experiments. The inset shows the experimentally determined specific heat (bold line) and the theoretical prediction for a finite number of ideal bosons in a harmonic trap. (From J. R. Ensher et al. (1996), with permission.)
A related work was performed at MIT on trapped Na atoms [48]. The dependence of the condensate fraction on temperature and the interaction energy as a function of the number of atoms in the condensate were investigated. The study of the effect of interaction energy was made possible by the high density of atoms achieved in the new "cloverleaf" trap. The measured interaction energy per atom showed a good agreement with the $N^{2/5}$ behaviour expected from the theory.

3.4. Study of collective excitations and propagation of sound in a Bose-Einstein condensate. - The collective excitations of the Bose-Einstein condensate have been studied both by the JILA group and by the MIT group. The excitation of the lowest collective modes of the condensates in the trap was achieved by a time-dependent perturbation of the magnetic trap potential. For a noninteracting gas, all modes have frequencies which are integer multiples of the trap frequency. Deviations from this behaviour are expected as an effect of interatomic interactions. In the mean-field picture, the interaction term in the Gross-Pitaevskii equation depends on $N a \sqrt{\nu}$, where $N$ is the number of atoms, $a$ is the scattering length and $\nu$ is the trap frequency. The normal modes of the condensate can be classified by a quantum number $m$ that gives the angular momentum projection in the direction of the trap symmetry axis. In [73] the $m = 0$ and $m = 2$ modes of a dilute condensate of $^{87}$Rb were investigated and their frequency measured as a function of the interaction strength. The $m = 0$ mode corresponds to an oscillation in radial size. The $m = 2$ mode resembles an ellipse whose major axis rotates in the plane orthogonal to the trap axis (fig. 17). The measured frequencies for the two excitation modes as a function of the interaction strength are shown in fig. 18. In the figure, the experimental data are compared with the theoretical predictions based on mean-field theory. The measured frequencies deviate from the harmonic trap values showing a fairly good agreement with theoretical calculations.

A similar experiment was performed by the MIT group on a sodium condensate. They studied $m = 0$ oscillations. The measured frequency 30.0(2) Hz agreed with the theoretical prediction $\sqrt{5/2} \nu z$ [74]. The oscillations observed in this experiment showed a damping with a decay time, for a nearly pure condensate, of 250(40) ms for the oscillation at $\sim 30$ Hz. Measurements on a thermal cloud ($T/T_c \approx 2$) gave a damping time of 80 ms for oscillations at a frequency of 35(4) Hz = 1.2(2)$\nu z$.

The dependence of the damping rate and of the oscillation frequencies on the sample temperature was studied in [75, 76]. In particular, temperatures were investigated for which both condensate and noncondensate atoms are present, a regime which is not well understood theoretically. In [75], the $m = 0$ and $m = 2$ modes were studied with the
same technique used in [73]. The main results of this work can be summarized as follows (fig. 19): A temperature-dependent shift of the excitation frequencies of the condensate can be observed, with the two modes shifting in opposite directions. In addition, for the \( m = 0 \) mode, a sharp feature in the temperature dependence is observed at \( T \approx 0.62 \, T_c \). This feature is not observed for the \( m = 2 \) mode. For both the \( m = 0 \) and \( m = 2 \) condensate excitations the damping rate quickly decreases with decreasing temperature. The two decay rates show the same behaviour. Interestingly, at temperatures where both condensed and noncondensed atoms are present, the condensate excitations are observed to decay more rapidly than the ones of noncondensed atoms. In the MIT experiment [76], collective excitations of the Bose gas at nonzero temperatures were studied for large condensates and in conditions corresponding to the hydrodynamic limit. The hydrodynamic oscillation of the thermal cloud, corresponding to the first sound, was observed. Also, an antisymmetric oscillation of the thermal cloud and the condensate, analogous to the out-of-phase second sound mode in liquid helium was identified. The results of this work cannot be completely explained with presently existing models and will certainly require a deeper investigation.

The propagation of sound in a dilute Bose-Einstein condensate was studied in [65]. Density perturbations were excited using the dipole force of a focused, blue-detuned laser beam. The propagation of sound waves was observed by recording rapid sequences of nondestructive phase contrast images.

The speed of sound \( c(r) \) is given by [77-79]

\[
(37) \quad c(r) = \sqrt{\frac{n(r)U}{2m}},
\]

where \( n(r) \) is the local density, \( m \) is the mass of the particles and \( U = 4\pi\hbar^2a/m \) gives the interaction of bosons with scattering length \( a \). The basic idea of the experiment reported in [65] is schematically shown in fig. 20. A condensate was formed, as in the other experiments described above, with the atoms confined in a magnetic trap. At time \( t = 0 \), the blue-detuned laser beam was switched on or off. The beam was focused at the center of the trap. The wavelength used (514 nm) was far enough from the sodium resonance...
at 589 nm to make heating from spontaneous emission negligible. If the laser beam was switched on after producing the condensate, the effect was to push the atoms out from the center of the trap creating two density peaks that propagated outward. If the focused laser beam was kept on during the evaporative cooling and switched off after the formation of the condensate, negative density perturbations were generated propagating outward with the same speed. The speed of sound was determined by measuring the position of the density peaks as a function of time. The resulting value was in good agreement with the value expected from eq. (37) (fig. 21).

3.5. Condensation of atoms with attractive interactions. - The experiments that first clearly showed BEC in dilute gases were performed on $^{87}$Rb [8] and $^{23}$Na [9]. Both these atomic species have a positive s-wave scattering length $a_s$, indicating that for cold atoms the interatomic interaction is repulsive. Evidence for Bose-Einstein condensation of $^7$Li, an atom for which $a_s$ is known to be negative, was first published in [10] and later confirmed in [11] with more accurate values for the number of condensate atoms. In the mean-field theory, the interaction energy is given by $U = 4\pi \hbar^2 a_s n / m$, where $n$ is the density and $m$ is the mass of the atoms. If $a_s < 0$, the energy decreases with increasing $n$, so the condensate is not stable and tends to collapse. For atoms in a confining potential,
Fig. 20. – Scheme for the excitation of positive (a) or negative (b) density perturbations in a condensate using a focused, blue-detuned laser beam. The perturbations propagate at the speed of sound. (From M. R. Andrews et al. (1997), with permission.)

recent theories predicted, however, that the kinetic zero-point energy can balance the attractive interaction and lead to a metastable condensate with only a small number of atoms in the condensate [80, 81].

The experiment reported in [11] gave results in agreement with theoretical predictions. As discussed in subsect. 3°2, a phase-contrast technique was used to observe the atoms directly in the trap. In this experiment, a permanent-magnet trap was used, so that only in situ imaging was possible. For temperatures between 120 and 330 nK, the maximum condensate number was found to be between 650 and 1300. This result is consistent with what is expected from theory for the trap used in this experiment. Work is actually in

Fig. 21. – Speed of sound vs. condensate peak density. The solid line is the theoretical prediction (From M. R. Andrews et al. (1997), with permission.)
progress to study the effect of attractive interactions on BEC also for other atoms, such as $^{85}$Rb, for which the s-wave scattering length is predicted to be negative.

A related subject under investigation is the observation of the so-called Feshbach resonances [82] in the scattering length of alkali atoms [83]. Such resonances could allow to control the magnitude or even the sign of the scattering length by applying external magnetic, optical or radio-frequency fields. Feshbach resonances were recently observed in an experiment performed at MIT on Bose-Einstein condensates of Na atoms [84]. A key ingredient in this experiment was the possibility of confining the condensate in an optical dipole trap [85]. It was predicted indeed that Feshbach resonances in Na would be observed for a relatively small applied magnetic field only for atoms in high-magnetic-field seeking states. Since these atoms cannot be trapped in magnetic traps, they were confined using the optical trap. The occurrence of the resonances was detected by two methods: The enhancement in the rate of inelastic collisions was detected as an increase of the losses of atoms from the trap as the magnetic field was swept across the resonance. Also, a clear evidence for a dispersive variation of the scattering length around the resonance was obtained by measuring the interaction energy for the trapped condensate vs. magnetic field by the time-of-flight absorption imaging method.

3.6. Production of two condensates by sympathetic cooling. – In [51], a cooling method called sympathetic cooling was used to produce two overlapping condensates made of $^{87}$Rb atoms in two different spin states. In sympathetic cooling, an atomic species is cooled via collisions with another species at a lower temperature. This method was first developed to cool ions in ion traps [86]. It had never been used before to cool neutral atoms, where interactions are in general weaker. In the experiment reported in [51], the two species corresponded to $^{87}$Rb atoms in the $| F = 1, m = -1 \rangle$ and $| F = 2, m = 2 \rangle$ spin states. Trapped atoms in the $| F = 1, m = -1 \rangle$ state were cooled, by evaporative cooling, down to Bose-Einstein condensation as in other experiments described above. In this case, however, atoms in the other spin state were left in the trap and observed at the end of the cooling process. Figure 22 shows the absorption image recorded by probing the atoms in $F = 2$ and the atoms in $F = 1$. In this case, the trap axis was slightly tilted with respect to the horizontal plane so that gravity produced a separation of the two
condensates. With a well-aligned trap, however, nearly perfectly overlapping condensates were obtained. Because of the different magnetic moments, evaporative cooling acts preferentially on the atoms in the $|F = 1, m = -1\rangle$ state which are less tightly confined in the trap. Therefore the evaporative cooling process leaves the initial number of atoms in the $|F = 2, m = 2\rangle$ state nearly unchanged (fig. 23). The study of the behaviour of the condensates provided important information on collisional parameters. By measuring the densities and loss rates for the atoms in the two states for various overlapping conditions, it was possible to determine the rate constant for binary inelastic collisions such as spin-exchange collisions. Another interesting result came from the observation that in the presence of the two condensates, the $|F = 2, m = 2\rangle$ condensate was displaced upward from its equilibrium position because of interaction with the $|F = 1, m = -1\rangle$ condensed atoms. This provided an indication that the interaction between the two condensates is repulsive.

The demonstration of the possibility of cooling a sample of atoms down to quantum degeneracy using sympathetic cooling is of great interest because of possible applications to other species. In particular, sympathetic cooling might allow the achievement of quantum degeneracy conditions in a gas of fermionic atoms. For spin-polarized fermionic atoms, in fact, simple evaporative cooling is not efficient because $s$-wave collisions are not possible for atoms in the same state. Also, the two fermionic atoms for which experiments are presently in progress, namely $^6$Li [87] and $^{40}$K [88], have a low natural abundance so that it is important to use a cooling technique which does not require a large initial number of atoms. More generally, sympathetic cooling may enable the achievement of Bose-Einstein condensation of atoms for which evaporative cooling does not work efficiently.

3'7. The coherence properties of the condensate and the "atom laser". – A gas of bosons all occupying the same state is described by a macroscopic wave function that is the solution of the nonlinear Schrödinger equation. Therefore, a Bose condensate should show coherence properties analogous to the ones of laser light. Such properties were investigated experimentally starting with the most striking effect, that is interference of the "matter waves" of two Bose condensates [89]. In this experiment, Na atoms were confined in a trap created by focusing a blue-detuned light sheet at the center of a "cloverleaf"
magnetic trap. The combination of magnetic force and optical dipole force produced a double-well potential. After evaporative cooling, two separate Bose condensates were produced instead of the single cigar-shaped condensate usually produced in the simple magnetic trap. The distance between the condensates could be varied by changing the intensity of the laser light. The overlapping of the two condensates was obtained by switching off the trap and letting the condensates expand. The interference pattern was observed by absorption imaging. In order to increase the spatial resolution, only a thin slice of the cloud was observed. This was achieved by first optically pumping the atoms in the relevant part of the cloud into a given hyperfine state using a thin sheet of light and then probing the atoms in this state with a pulse of resonant light. Figure 24 shows the interference of two condensates as observed, after 40 ms of expansion, for two different intensities of the blue-detuned laser light in the trap. Nearly straight fringes can be observed with a contrast that, after calibration, was found to be in the range 50%-100%. The measured spacing between the fringes, ~ 20 μm and ~ 15 μm, respectively, was consistent with the value of the de Broglie wavelength \( \lambda \) associated with the relative motion of the atoms

\[
\lambda = \frac{ht}{md},
\]

where \( h \) is Planck's constant, \( m \) is the atomic mass, \( d \sim 30-40 \mu m \) was the initial spatial separation between the condensates and \( t = 40 \) ms was the expansion time before the observation. The slight curvature of the fringes in fig. 24a was attributed to the effect of
interactions which becomes nonnegligible for small separations of the two condensates. The main result of this work was to demonstrate the first-order coherence of a Bose condensate and long-range correlations over the extent of the condensate. The methods developed allowed for the first time to address experimentally the debated question of the phase of a Bose-Einstein condensate showing the existence of such a phase and the possibility of comparing the phases of two condensates.

Higher-order coherence properties of the condensate were studied in [90]. Starting from the analogy between the intensity fluctuations in a light beam and the density in an atomic sample, information on statistical correlations in a condensate was obtained by studying three-body recombination rates. Indeed, in [91] it was calculated that the rate of three-body recombination in a condensate for noninteracting atoms should be reduced by a factor 3! with respect to the corresponding rate in a thermal cloud with the same mean density. In the experiment reported in [90], collision rates were inferred from the loss rate of atoms from the trap. Three-body recombination processes could be distinguished from other loss processes by measuring the loss rate as a function of density. By comparing the data obtained for noncondensed and condensed samples, it was found that the three-body recombination rate was smaller in the case of a condensate by a factor 7.4 (2.0). The measured value is then in agreement with the theoretical value of 3! and demonstrates that density fluctuations are reduced in a condensate relative to a thermal gas. In the language of quantum optics, condensate atoms appear to be less bunched than thermal atoms.

Several papers have recently discussed the analogies between coherent matter and coherent light [92-96]. One of the most interesting prospects in this field is the realization of an “atom laser”. In fact, atoms in a magnetic trap can be considered as analogous to photons in an optical cavity. The thermal cloud of ultracold atoms plays the role of the active medium and, in the experiments performed so far, evaporative cooling acts as the “excitation” mechanism which makes the atoms accumulate into a single state of the trap. It is obvious that, contrary to the photon case, the number of atoms cannot be amplified. However, the enhanced probability for bosons to occupy an already occupied state of the system can be seen as the analogue of an amplification mechanism. A key element in the realization of the “atom laser” is the possibility of coupling out particles in a coherent way. The simplest way to couple atoms out of the condensate is to switch off the trapping potential and let the atoms fall under the effect of gravity. A more controlled method was demonstrated in [97]. Rf radiation was used to create Bose condensates in a superposition of trapped and untrapped states. After producing a condensate of $5 \times 10^6$ sodium atoms in the $F = 1, m_F = -1$ ground state, a resonant rf magnetic field was switched on with a polarization orthogonal to the magnetic field at the trap center. This coupled a part of the trapped atoms into the $m_F = 0$ and into the $m_F = 1$ states. Atoms in the $m_F = 1$ state were repelled from the trap while the untrapped $m_F = 0$ atoms expanded freely while moving downward under the effect of gravity. The amplitude of the rf pulse could be adjusted to get an out-coupling of atoms between 0% and 100%. Interestingly, by varying the intensity of the rf pulse, Rabi oscillations were observed with the same Rabi frequency as for the single atom. Using subsequent rf pulses, multiple pulses of atoms out of the condensate were obtained (fig. 25). In the same work, another output coupler scheme was demonstrated using a nonadiabatic rf sweep through resonance to transfer atoms into the untrapped states. This scheme is less sensitive than the other to changes in the bias magnetic field of the trap. In the same work, the possibility of taking advantage of Majorana spin-flips as an out-coupling mechanism was also shown.

In [89], a test of the coherence of the pulses of atoms was performed. Pulses from two
Fig. 25. – Atom laser. The figure shows successive pulses of coherent sodium atoms coupled out from a Bose-Einstein condensate. (Figure courtesy of W. Ketterle.)

separate condensates were combined and high-contrast interference fringes were observed. This completed the proof that a Bose condensate with an out-coupling mechanism such as the one just described can be considered as a realization of an “atom laser”.
4. Conclusions and future prospects

At the time of this writing, almost three years have passed since the first observation of BEC in atomic gases. The laboratories where BEC was first achieved had the possibility of investigating a wide range of interesting phenomena some of which had been predicted theoretically but never studied experimentally in such simple and well-controlled systems. During this time, several laboratories in the world have been trying to reproduce the experiments and to achieve BEC.

Recently, a few laboratories announced the observation of BEC: D. Heinzen's group at the University of Texas at Austin observed BEC of $^{87}$Rb atoms using a Zeeman-slowed atomic beam and evaporative cooling of the atoms in a TOP magnetic trap. L. Hau and collaborators at the Rowland Institute in Cambridge, MA, used a Zeeman-slowed atomic beam of Na and a Ioffe trap. The group of M. Kasevich, at Stanford University, achieved BEC of $^{87}$Rb atoms with a single vapor cell and a TOP trap. At the University of Konstanz in Germany, the group of G. Rempe observed BEC of $^{87}$Rb atoms in a double MOT apparatus with a Ioffe-type magnetic trap. In the laboratory of T. Hänsch at the University of Munich/MPI for Quantum Optics, BEC of $^{87}$Rb was achieved using a double-MOT apparatus and a Ioffe-type magnetic trap. The group of W. Phillips at NIST, Gaithersburg achieved BEC with sodium using a Zeeman-cooled atomic beam and a TOP trap. It is interesting to notice that in this experiment BEC was reached also by evaporating only with the "circle of death" of the TOP trap. At the Ecole Normale Superieure in Paris, J. Dalibard and colleagues observed BEC of rubidium atoms using an apparatus based on a double-MOT and a Ioffe-type magnetic trap.

The experimental results also stimulated a lot of theoretical work and several papers were published interpreting the new observations and proposing new possible experiments.

Amongst the future challenges one can envisage the achievement of BEC with other atoms. Of particular interest are H and He*. Work is also in progress on K [49], Cs [52,98], and even on molecules [99].

The effect of interactions needs to be further investigated. The case of atoms with attractive interactions and the possibility of modifying the strength of the interaction by external fields are certainly important subjects to be further explored.

There are important phenomena that have been predicted theoretically and observed in other systems. Notable examples are vortices and second sound that have been widely investigated in superfluid helium. The study of these phenomena in the well-controlled conditions of experiments with dilute atomic gases will provide important information.

The possibility of manipulating the condensates should allow studies of tunnelling effects between two condensates and the observation of Josephson-type phenomena.

The future possibilities of atom lasers can be imagined considering the major impact that optical lasers had in several fields.

Finally, one of the most interesting prospects is to investigate the properties of an ultracold gas of fermionic atoms. At phase space densities similar to the ones achieved in BEC experiments, the behaviour of a degenerate Fermi gas may be studied and, at still lower temperatures, the superfluid phase transition could be observed [100-103]. Promising candidates which are presently investigated are $^{6}$Li [87] and $^{40}$K [88].

Note added in proofs. - After the completion of this review, many interesting papers appeared reporting new results in the investigation of BEC. An extensive review of the theory of BEC in trapped gases has been published in [104]. A discussion of recent
Theoretical and experimental results can be found in the proceedings of the E. Fermi School on "Bose-Einstein condensation in atomic gases" held in Varenna in the summer 1998 [105]. During that School, the first evidence of BEC in atomic hydrogen was presented [106].

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