Collective excitations of a ^{87}Rb Bose condensate in the Thomas-Fermi regime

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Abstract. – Collective excitations have been observed in a ^{87}Rb Bose condensate, confined in a magneto-static, elongated, harmonic trap. We have measured the frequency of two different modes with zero angular momentum along the symmetry axis of the trap. Both the frequencies are in good agreement with the values predicted by the hydrodynamic theory of superfluids holding in the Thomas-Fermi regime.

One of the most attractive features of Bose-Einstein condensation (BEC) in dilute atomic vapours can be certainly traced to the possibility of making quantitative comparison with well-developed many-body theories. Since the first experimental realizations of Bose-Einstein condensation [1], several physical quantities have been measured and found in satisfactory agreement with theory: ground-state occupation, critical temperature, specific heat, frequency and damping of collective excitations, propagation of sound, etc. [2]. Among these, frequency measurements deserve a special attention, as they are usually superior in precision and accuracy. In addition, the spectrum of elementary excitations provides information on the dynamics of the condensate and represents a more sensitive test for theories than the thermodynamics quantities, that depend on the sum over energy states. Indeed, several authors have calculated the eigenfrequencies for a harmonically trapped condensate [3].

Excitations can be experimentally investigated by suitably perturbing the condensate and then observing the oscillations of its density distribution. At low frequencies $\hbar\omega \ll \mu$, where μ is the chemical potential, the wavelength λ of the perturbation is comparable to the size of the condensate: therefore, if the ground state lies in the Thomas-Fermi (TF) regime, the quantum pressure term associated with the kinetic energy can be safely neglected. Within this approximation, the eigenfrequencies of the condensate have been analytically derived by

Stringari for a spherical trap. For axially symmetric traps, analytical solutions are available only for a few modes [4].

For a spherical harmonic trap, the condensate density perturbation can be decomposed in a set of eigenfunctions of the type $P_\ell^{(2n)}(r)r^\ell Y_{\ell m}(\theta, \phi)$, where $P_\ell^{(2n)}(r)$ are polynomials of degree $2n$. Consequently, three quantum numbers (n, ℓ, m) are used to label the collective excitations, with ℓ, m indicating the eigenvalues of the total angular momentum and of its z -component. For an elliptical trap, only the component along the z symmetry axis is conserved. In particular, two low-frequency modes with $m = 0$ are linear superpositions of the monopole $(1, 0, 0)$ and quadrupole $(0, 2, 0)$ modes of the spherical trap. For the spherical trap, the monopole mode is just an oscillation rescaling the wave function of the condensate; in the quadrupole mode, the condensate elongates along the z -axis while shrinking in the radial direction, and viceversa. In our trap, since the two modes mix, we expect to observe both as oscillations of the condensate aspect ratio $\text{AR} = R_z/R_r$, where R_z and R_r are, respectively, the axial and radial radii. Finally, we remind that the sloshing motions of the condensate center-of-mass correspond to the dipole $(0, 1, m)$ modes of the spherical trap and occur at the harmonic oscillator frequencies.

So far only two groups have published experimental results on this subject: the JILA group working on rubidium in a TOP trap and the MIT group working on sodium in a cigar-shaped Ioffe trap. The group at JILA observed the lowest, monopole-quadrupole mixed, $m = 0$ excitation, the $(0, 2, 2)$ [5, 6] and a superposition of $m = 0$ modes [7]. The group at MIT investigated the two mixed $m = 0$ excitations [8, 9], and recently the surface modes $(0, \ell, \pm\ell)$ with $\ell = 2, 4$ [10]. This work extends the analysis of rubidium atoms measuring the frequency of the two mixed, axially symmetric modes in a cigar-shaped trap. With respect to the original apparatus of JILA, our experimental conditions allow to work in the Thomas-Fermi regime for which analytical theoretical predictions are available.

Experimental set-up. – Most of the apparatus derives from the double magneto-optical trap (MOT) set-up, that was devised for our previous work on potassium [11]. The trapping, pushing and probing light is derived from a single titanium:sapphire laser, tuned close to the $5^2S_{1/2} F=2 \rightarrow 5^2P_{3/2} F'=3$ transition, emitting 500 mW. The first, loading MOT (MOT1) is run with 150 mW of total power, split into three retroreflected beams with a diameter of 2 cm. For the second MOT (MOT2) we have six independent beams, each of 10 mW of power and 1 cm of diameter. A grating-stabilised diode laser provides 5 mW of repumping light for both MOTs.

To magnetically trap the atoms, we use the 4 coils shown in fig. 1, that give rise to a cigar-shaped Ioffe magnetic potential elongated along the z symmetry axis. Our design is inspired by the scheme first introduced in [12] and is operated with high currents. All the coils are made of 1/8-inch, water-cooled, copper tube. The three identical coils consist of 15 windings with diameters ranging from 3 cm to 6 cm. The fourth coil is composed of 6 windings with a diameter of 12 cm. The two coils Q1 and Q2 generate a quadrupole field symmetric around the vertical y -axis, along which the field gradient has been measured to be 10 mT/m·A. Curvature coil C and antibias coil A produce opposite fields along the z -axis. The modulus of the overall magnetic field has a minimum displaced by 5 mm from the center of the quadrupole field towards the curvature coil: the axial field curvature is 0.46 T/A·m².

With 240 A running in all the coils, we expect the harmonic oscillator axial frequency to be $\nu_z = 13$ Hz, for atoms in the $F = 2$, $m_F = 2$ state. The radial frequency ν_r can be adjusted by tuning $B_b = \min(|\mathbf{B}|)$, the field at the bottom of the trap, resulting in calculated value of $\nu_r = (2.18/\sqrt{B_b[\text{T}]})$ Hz. Typical operating values for B_b range from 0.14 mT to 0.18 mT. We note here that we have observed slow 2% variations of the bottom of the trapping field,

Fig. 1 – Our four-coil magnetic trap. Coils Q1 and Q2 are in anti-Helmholtz configuration and provide quadrupole field in the vertical direction also used during the MOT operation. Coil C provides the curvature along the symmetry axes z of the magnetic trap, and A almost cancels the bias fields of the curvature coil C.

this is probably caused by mechanical instabilities due to the heat generated by the dissipated power of 1.8 kW. A set of additional three pairs of Helmholtz coils provides control of stray magnetic fields (shim coils).

We load $1.5 \cdot 10^9$ atoms in MOT1 within a few seconds, $5 \cdot 10^8$ of which are pushed, twice in a second, and magnetically guided towards MOT2, that recaptures about 30% of them. The MOT2 quadrupole field is generated by coils Q1 and Q2: the gradient along the y -axis is 0.15 T/m. With 50 shots we typically load $1.2 \cdot 10^9$ atoms in MOT2. Then, we increase the atomic density with 30 ms of CMOT [13] and cool by means of a 8 ms long, molasses phase.

To magnetically confine the atoms, it is highly desirable to have a spin-polarized sample to start with. We shine a 200 μ s pulse of σ -polarized light along the z -axis in resonance with the $F = 2 \rightarrow F' = 2$ transition, overlapped by the repumping light, which is actually switched off 200 μ s later to empty the $F = 1$ ground level.

As the MOT (centered at the center of the quadrupole field) and the minimum of the harmonic magnetic trap are 5 mm apart, the transfer of atoms in the magnetic trap has to be done in few steps. We first load the atoms in a quadrupole field with a gradient of 0.7 T/m, roughly corresponding to “mode-matching” condition, *i.e.* the acquired magnetic potential energy equals the kinetic energy, which ensures minimum losses in the phase-space density. Then we adiabatically increase the gradient to 2.4 T/m by ramping the current in 400 ms to its maximum value, 240 A. Finally, the curvature (C) and anti-bias (A) coils are smoothly (in 750 ms) switched in a series configuration with the quadrupole ones, and the final configuration of harmonic potential is reached.

At the end of this loading procedure, 30% of atoms are transferred from the MOT to the harmonic magnetic trap and we start rf forced evaporation with $4 \cdot 10^8$ atoms at 500 μ K. The rf frequency is first ramped for 20 s with an exponential law from 20 MHz to a value which is only 100 kHz above the trap bottom frequency $\nu_{\text{rf}}^0 = \mu_0 B_b / 2h$. Afterwards, a 5 s long, linear ramp takes the rf close to ν_{rf}^0 : the phase transition takes place within a range of 5 kHz above ν_{rf}^0 . We estimate the initial elastic collision rate to be $\gamma = 30 \text{ s}^{-1}$, that, combined with the

Fig. 2 – Monopole-quadrupole mixed collective excitations $m = 0$ at 20 Hz (a) and 330 Hz (b). The aspect ratio of the expanded cloud is plotted *vs.* the time interval between excitation and switching off of the trap.

60 s trap lifetime, gives a ratio of “good” to “bad” collisions rates of 1800.

To determine the number of atoms, their density and temperature, we rely on absorption imaging. Atoms are released from the trap and let fall by switching off the coils current in 1 ms. After a delay of 10 to 25 ms, we flash for 150 μs a probing beam at one tenth of the saturation intensity, resonant with the $F = 2 \rightarrow F' = 3$ transition. The shadow cast by the cloud is imaged with two lenses on a CCD camera. The resolution limit due to the CCD pixel size (24 μs) combined with the magnification of 6, is smaller than the 7 μm diffraction limit of the first lens ($f = 60$ mm, NA=0.28). We process three images to obtain the two-dimensional column density $\tilde{n}(y, z) = \int n(x, y, z) dx$, which is then fitted assuming that $n(x, y, z)$ is simply the sum of a Gaussian distribution (uncondensed fraction) and the inverted parabola, solution of the Gross-Pitaevskii equation in the TF approximation (condensed fraction). The effect of free expansion, which is trivial for the Gaussian part, is taken into account also for the condensate as a rescaling of the radii, according to [14]. The temperature is read from the Gaussian widths of the thermal cloud.

We observed BEC transition at a temperature $T_c = 200$ nK with $2 \cdot 10^5$ atoms, the peak density n_c being $7 \cdot 10^{19} \text{ m}^{-3}$. The number of condensed atoms shows fluctuations of 20% from run to run attributed to thermal effects on the magnetic trap coils. Given our imaging sensitivity, we estimate that the fraction of the thermal atoms is always less than 50%. This first analysis of the thermal component provides an upper limit of $0.8 T_c$ for the temperature of the gas.

For the TF approximation to hold, one needs $\eta = Na/a_{\text{ho}} \gg 1$, where $a = 5.8$ nm is the s -wave, triplet, scattering length and $a_{\text{ho}} = \sqrt{\hbar/m\omega_{\text{ho}}}$, with $\omega_{\text{ho}}^3 = \omega_x\omega_y\omega_z$, is the length of the harmonic oscillator. During the oscillations measurements described below, we had an average number of condensed atoms of $5 \cdot 10^4$, which combined with the observed frequencies yields $\eta \sim 200$: this is about a factor 5 larger than the values reported for rubidium in [5] and allows to investigate the strong interacting limit [4].

Oscillations. – We excite the collective oscillations of the condensate by simply modulating the trap bias field with the shim coils in the z -direction. This allows us to change the radial but not the axial frequency and only $m = 0$ modes can be excited. In the TF approximation, the frequencies of the two $m = 0$ modes are derived analytically [4]:

$$\nu_{1,2}^2 = \nu_r^2 \left(2 + \frac{3}{2}\lambda^2 \mp \frac{1}{2}\sqrt{9\lambda^4 - 16\lambda^2 + 16} \right),$$

with $\lambda = \nu_z/\nu_r$. In our experiment $\lambda = 1/13$, thus the two frequencies can be approximated by the $\lambda \rightarrow 0$ limit values: $\sqrt{5/2}\nu_z$ and $2\nu_r$, with an error of about 4 parts in 10^4 .

A few hundreds of milliseconds after the condensate has formed, we feed five cycles of a pure sine modulation to the z -axis shim coils current and wait for an interval Δt before switching off the field and performing absorption imaging. The amplitude of the modulation on the bias field ranges from 4 to 28 μT , *i.e.* from 2% to 15% of the static value. Oscillations are observed in the radial and axial radii of the expanded condensate as Δt is changed. We stress that our detection method is destructive: each run yields one single data point. Since both the radial and axial widths scale as $N^{1/5}$, we reduce the noise brought by fluctuations of N by plotting the aspect ratio AR of the cloud instead of the widths R_z , R_r separately.

To calibrate the axial frequency, we observe the sloshing motion of the thermal cloud center-of-mass caused by a sudden perturbation of the magnetic field: we have measured $\nu_z = 12.6(2)$ Hz. The collective excitation ν_1 is observed after modulating at 20 Hz: the fit shown in fig. 2a yields $\nu_1 = 19.9(1)$ Hz. The ratio ν_1/ν_z equals 1.58(3), in good agreement with its predicted value of 1.581.

The radial trap frequency has been determined by observing the sloshing motion of the condensate. The measured value is $\nu_r = 166(2)$ Hz with a bias field of 178(1) μT . This value agrees with that yielded by the “parametric heating”: we have modulated the radial frequency, as usual via the bias field, and observed the temperature of the thermal cloud as a function of the modulation frequency. Resonant parametric heating shows as 6-fold increase of the temperature, from 1.4 μK to 8 μK . From this method we deduce $\nu_r = 164(1)$ Hz.

The condensate collective oscillation shown in fig. 2b has been excited by modulating at 335 Hz the 187(1) μT static bias field; its frequency is $\nu_2 = 330(1)$ Hz. The ratio $\nu_2/\nu_r = 2.06(2)$ fairly agrees with the predicted value of 2.

We also measured the damping rate of the two modes: $\tau = 0.9(4)$ s for the collective excitation at $\sqrt{5/2}\nu_z$ and $\tau=0.14(4)$ s for the mode at $2\nu_r$. Considerable theoretical work has been done to predict the damping of collective excitations and its temperature dependence. These predictions can be so far compared only with experimental results coming from the JILA [6] and the MIT [9] groups. On this point it would be interesting to extend the present results. The level of accuracy and reproducibility demonstrated by our experimental apparatus will allow future investigations of not yet fully clarified aspects such as the damping phenomena, non-linear behaviours and finite-size effects for the axial and radial modes.

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