Precision Measurement of Gravity with Cold Atoms in an Optical Lattice and Comparison with a Classical Gravimeter

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We report on a precision measurement of gravitational acceleration using ultracold strontium atoms confined in an amplitude-modulated vertical optical lattice. An uncertainty $\Delta g/g = 10^{-7}$ is reached by measuring at the 5th harmonic of the Bloch frequency. The value obtained with this microscopic quantum system is consistent with the one measured with a classical gravimeter. Using lattice modulation to prepare the atomic sample, we also achieve high visibility of Bloch oscillations for $\sim 20$ s. These results can be of relevance for testing gravitational redshift and Newtonian law at micrometer scale.

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Atom interferometry, and in general methods based on quantum interference of ultracold atoms, were largely used in recent years for gravitational physics experiments and new exciting prospects can be envisioned in the near future [1]. For example, Raman interferometry was used for precise measurements of Earth’s gravitational acceleration $g$ [2] and its gradient [3], for determining the value of the gravitational constant [4,5], for a possible redefinition of the kg [6], and for geophysical applications [7]. Schemes based on Bloch oscillations of atoms trapped in vertical optical lattices were also used to measure gravity with the possibility of combining high sensitivity and micrometric spatial resolution [8–10]. The results of atom interferometry experiments were interpreted as tests of the isotropy of post-Newtonian gravity [11], of quantum gravity [12], and of gravitational redshift [13]. Prospects include high precision tests of the weak equivalence principle [14,15], the detection of gravitational waves [16,17], and future experiments in space [18].

Here, we present a precision measurement of gravitational acceleration $g$ with a method based on ultracold $^{88}$Sr atoms confined in an amplitude-modulated vertical optical lattice [19] and compare the result with the value obtained with a classical absolute gravimeter based on an optical interferometer with one arm including a freely falling corner cube. By careful analysis of systematics effects, we improved the accuracy of our previous gravity determination [9,20] by more than 2 orders of magnitude. Using amplitude modulation of the lattice to prepare the atomic sample, we also achieved high visibility of the Bloch oscillations for $\sim 20$ s. We discuss the precision of the two methods for the determination of $g$. The use of atoms confined in a volume of a few micrometers size makes this experiment qualitatively different from similar, higher precision experiments performed with freely falling atoms [2,21]. For example, our data can be interpreted as a measurement of the gravitational redshift to the Compton frequency of Sr matter waves, as suggested by Müller et al. [13]. While the interpretation of atom interferometer redshift tests is complicated by special relativistic time dilation if the atoms are moving [22,23], Bloch oscillations experiments with stationary lattices provide a measurement of the purely gravitational effect.

The experimental setup is based on cooled and trapped $^{88}$Sr atoms [9] (Fig. 1). Atoms from a thermal beam are slowed in a Zeeman slower and trapped in a “blue” magneto-optical trap (MOT) operating on the $^1S_0^{-1}P_1$ resonance transition at 461 nm. The temperature is further reduced by a second cooling stage in a “red” MOT operating on the $^1S_0^{-3}P_1$ intercombination transition at 689 nm. This produces about $10^6$ atoms at a temperature of 0.6 $\mu$K. Since the force of gravity is comparable to the force produced by the red MOT on the atoms, the cloud of trapped atoms assumes a disklike shape with a vertical size of $27 \mu$m and a radial size of $180 \mu$m. The atoms are adiabatically loaded in an optical lattice in 300 $\mu$s. The lattice is generated by a single-mode frequency-doubled Nd:YVO$_4$ laser ($\lambda_{\text{L}} = 532$ nm) delivering up to 1 W on the

FIG. 1 (color online). Experimental setup for the measurement of gravity with $^{88}$Sr atoms trapped in a vertical optical lattice and the comparison with a classical absolute gravimeter (FG5). AM: amplitude-modulation signal; AOM: acousto-optical modulator; FCC: falling corner cube; ID: interference detector; PD: photodiode; PMF: polarization-maintaining fiber; RCC: reference corner cube.
atoms with a beam waist of 557(7) μm. The beam is vertically aligned and retroreflected by a mirror. The atomic sample in the lattice has a vertical rms size of 14 μm and a horizontal size of 100 μm. The Bloch frequency is \( v_B = m_{Sr} g \lambda_L / 2h \approx 574.3 \text{ Hz} \) where \( m_{Sr} \) is the mass of \(^{88}\text{Sr}\) atoms and \( h \) is Planck constant. The corresponding lattice photon recoil energy is \( E_R = 8 \text{ kHz} \times h \). In typical conditions the lattice depth ranges from 2.3 to 3 \( E_R \). While the energy gap \( E_G \) at the recoil momentum \( k_L \) is \( E_G \approx E_R \). The width of the first energy band in the lattice potential is about 0.5 \( E_G \). Landau-Zener tunneling is negligible in these conditions. The lattice depth is stabilized by a servo loop acting on the rf signal driving an acousto-optical modulator (AOM). The same AOM is used to add an amplitude modulation to the lattice potential. The atomic cloud can be imaged either in situ or with the usual time-of-flight technique using resonant absorption imaging on a CCD camera with a spatial resolution of 5 μm.

After loading \(^{88}\text{Sr}\) atoms in the vertical lattice, the trap depth is modulated sinusoidally. When the modulation frequency matches an integer harmonic of the Bloch frequency, atoms start to tunnel in neighboring lattice sites giving rise to a net increase in the vertical spatial atomic distribution which is observed in situ by resonant absorption imaging (Fig. 2). In this experiment, the modulation at a frequency \( v_m = 5 \times v_B \) is applied for a time \( \sim 10 \text{ s} \) which is experimentally set as a compromise between the linear increase of resolution with time and the exponential decay of the signal due to the loss of atoms from the lattice. Because of the extremely small scattering length of \(^{88}\text{Sr}\), the effects of cold atoms collisions are negligible over this time scale. The amplitude-modulation depth is varied in the range 4%–10% in order to optimize the signal-to-noise ratio; the corresponding estimated tunneling rate is 3–6 s\(^{-1}\) [19]. In this interval, we do not observe any dependence of the resonance position and shape on the modulation depth. The recording time for a whole resonance spectrum is about 1 h and leads to a maximum resolution of \( 1.5 \times 10^{-3} \) for \( v_B \).

A number of sources of systematic error must be considered. Since the lattice laser is not frequency stabilized, a calibration of its frequency is required. To this purpose, part of the lattice laser light is sent to an optical frequency comb. The measured fluctuation of the laser frequency is \( \sim 100 \text{ MHz} \) on the relevant time scale of the experiment. The correction for the index of refraction of the Sr cloud and the background gas in the vacuum chamber is negligible. The vertical alignment of the lattice is checked with a precision of 0.2 mrad, corresponding to a relative uncertainty of \( 2 \times 10^{-8} \) on \( g \). Overlapping the downward laser beam with the reflection from the surface of water in a glass container inserted in the beam. A tiltmeter with a resolution of 1.7 μrad attached to the optical table is employed to check the alignment stability during the measurements. The potential resulting from the interference of the two Gaussian beams depends on their longitudinal intensity profile and their wave front curvatures, producing an inhomogeneity of the optical lattice in the axial direction [24]. This gives rise to two correction terms on \( g \), namely, \( \Delta g_U \) for the spatial derivative of the lattice potential and \( \Delta g_k \) for the spatial derivative of the difference between the Gouy phase of the two counterpropagating beams. The two terms are estimated by a precise determination of the geometry of the incoming and the reflected trapping beams and the position of the cloud with respect to the beam waist, with a relative uncertainty of 1%. An independent determination of the transverse beam size at the position of the atomic cloud is obtained by measuring the axial and radial atomic trap oscillation frequencies through parametric heating [25,26]. For typical experimental parameters, the two terms are \( \Delta g_U = 1.53(3) \times 10^{-5} \text{ m/s}^2 \) and \( \Delta g_k = 1.0(2) \times 10^{-8} \text{ m/s}^2 \). Tidal effects are evaluated and removed from the raw data using the same algorithm and potential model used for the absolute gravimeter data processing. The peak-to-peak effect of tides at our site is \( \sim 2 \times 10^{-6} \text{ m/s}^2 \). Since each measurement lasts about 1 h the variation of \( g \) during a single measurement due to tides is below \( 10^{-7} \text{ m/s}^2 \). The values of \( m_{Sr} \) and \( h \) are both known with a relative uncertainty of \( \sim 5 \times 10^{-8} \). All the other potential sources of systematic shifts that we evaluated (spurious higher harmonics of amplitude modulation, Bloch-Siegert shift [27], gravity due to surrounding masses, magnetic field gradients) are below the current accuracy level. Table I summarizes the main systematic shifts for the gravity measurements using the amplitude-modulation method. The values of individual shifts depend on the experimental conditions; the quoted uncertainties are typical values.

Figure 3 presents a set of 21 determinations of \( g \) with \(^{88}\text{Sr}\) atoms. The error bars result from the quadrature sum of the statistical errors coming from the fit of the amplitude-modulation resonance signal and the uncertainty on systematic corrections. The weighted mean of

![Figure 2](https://example.com/figure2.png)

**FIG. 2** (color online). Spectrum recorded by modulation of the lattice depth at the 5th harmonic of the Bloch frequency for 10.4 s with a modulation depth of 8%. The red line is a fit of experimental data with a sinc function.
gradient value of 14(5) cm together with the estimated vertical gravity from the atomic probe position. The difference in height is performed in the same laboratory at a distance of 1.15 m corner-cube (FG5, Micro-g LaCoste). The measurement interferometer with one arm including a freely falling is provided by an absolute gravimeter based on an optical t certainty corresponds to 1 standard deviation.

**FIG. 3 (color online).** Measurements of $g$ using the amplitude-modulation technique. Each experimental point is corrected for the systematic effects presented in Table I. The red dashed line represents the weighted mean of the 21 measurements. The blue solid line is the value obtained with the classical absolute FG5 gravimeter.

**TABLE I.** Systematic corrections and their associated uncertainties ($\times 10^{-7}$) for the gravity measurement with $^{88}$Sr atoms in the amplitude-modulated optical lattice.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Correction</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice wavelength</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Lattice beam vertical align.</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Stark shift (beam geometry)</td>
<td>14.3–17.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Experiment timing</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Tides</td>
<td>−1.4−0.9</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Height difference</td>
<td>4.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Refraction index</td>
<td>0</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>Fundamental constants</td>
<td>0</td>
<td>0.7</td>
</tr>
<tr>
<td>Systematics total</td>
<td>17.2–22.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>

our data is $g_{\text{atom}} = 9.8049232(14) \text{ m/s}^2$ where the uncertainty corresponds to 1 standard deviation.

The reference value for local gravitational acceleration is provided by an absolute gravimeter based on an optical interferometer with one arm including a freely falling corner-cube (FG5, Micro-g LaCoste). The measurement is performed in the same laboratory at a distance of 1.15 m from the atomic probe position. The difference in height of 14(5) cm together with the estimated vertical gravity gradient value $g_{zz} = -3.09 \times 10^{-6} \text{ s}^{-2}$ at the laboratory site is taken into account in the data analysis. The result is $g_{\text{fg5}} = 9.804921609(84) \text{ m/s}^2$.

The comparison of the value obtained with the quantum mechanical atomic sensor and the one obtained with the classical gravimeter shows that they agree within the experimental errors.

With minor modifications of the experimental procedure, in this work we also determine $g$ by measuring the frequency of the Bloch oscillations of the atoms in the vertical optical lattice. Because of a better vacuum and taking advantage of the lattice modulation method to reduce the initial momentum distribution of the atoms in the lattice [19], we considerably improve the visibility of the oscillations and, as a consequence, the frequency resolution compared with previous experiments [9]. After the transfer of the atoms in the vertical optical lattice, an amplitude-modulation burst with typical duration of 120 cycles at $\nu_{mol} = \nu_R$ is applied. The quantum phase of the atomic wave function induced by the amplitude modulation gives rise to an interference effect which results in an enhanced visibility of the Bloch oscillations peaks in the time-of-flight image of the atomic cloud [28]. After turning off the modulation, we let the atomic cloud evolve for a time $T$. Finally, we switch off the optical lattice within 5 $\mu$s to measure the momentum distribution of the atoms in ballistic expansion by taking an absorption picture with a CCD camera. In order to optimize the visibility through this quantum interference effect, we set the time of flight to 14 ms. As shown in Fig. 4, we observe Bloch oscillations with high visibility for $\sim 20$ s. From the fit of the mean atomic momentum we can estimate the Bloch frequency $\nu_B$ with $1.7 \times 10^{-7}$ statistical uncertainty. In comparison with the determination of $\nu_B$ obtained with the resonant amplitude-modulation technique, however, we find a considerably larger scattering in repeated measurements, mainly due to the initial position instability of the atomic trap and to a higher sensitivity to the timing of the experiment. The value for $g$ obtained with the Bloch oscillation technique is $g_{\text{Bloch}} = 9.80488(6) \text{ m/s}^2$, which is consistent with the measurement presented above but is affected by a larger relative uncertainty of $6 \times 10^{-6}$.

In conclusion, we have performed an accurate measurement of gravitational acceleration using ultracold $^{88}$Sr atoms confined in a vertical optical lattice. The result agrees within 140 ppb with the value obtained with a classical FG5 gravimeter. This result improves by 1 order of magnitude in sensitivity and by more than 2 in

**FIG. 4 (color online).** Long-lived Bloch oscillations for Sr atoms in the vertical lattice under the influence of gravity. Each picture shows one Bloch cycle in successive time-of-flight absorption images giving the momentum distribution at the time of release from the lattice. Displayed are the first (a), the 2900th (b), the 7500th (c), and the 9800th (d) Bloch cycles.
accuracy over previous results [9,20]. This provides a correspondingly improved test of the gravitational redshift in the frame of the model proposed in Ref. [13]. In this experiment, we also observe persistent Bloch oscillations with high visibility for \(\sim 20\) s but we find that as a method for the measurement of \(g\) it is affected by a larger uncertainty. We believe that the amplitude-modulation method for gravity measurements has the potential for further improvement. The present results are indeed mainly limited by frequency stability and wave front curvature of the lattice laser beam. The first effect can be reduced by using a frequency stabilized laser. The second could be reduced either by increasing the lattice beam waist or by using a blue-detuned lattice [24]. Application of Ramsey-like schemes with short separated modulation pulses [19] might increase the resolution and reduce systematic effects produced by the lattice. The results of this work are of interest also for experiments on Casimir-Polder effect [29], tests of Newtonian gravitational law at micrometer distances [10], and for precision measurements with nondestructive cavity QED techniques [30].

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